Ilkogretim Online - Elementary Education Online, 2021; Vol 20 (Issue 3): pp. 1997-2002
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2021.03.230

FIXED POINT THEORY IN HYPER CONVEX METRIC SPACES

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Abstract- The problem of extending uniformly continuous mappings between metric spaces was proposed by Aronszajn and Panitchpakdi in 1956, with respect to Hyperconvex. The structure offered by the hyperconvexity of the metric to space was evident from the very beginning. Due to the richness of this, hyperconvectic metric locations, notably in the late 1880's by pioneering works due to Baillon, Sine and Soardi, were developed to establish a very deep and exhaustive Fixed Point Theory. This theory refers both to single and multivalued mappings and to the best outcomes. In the last decade of metric fixed point theory on hyperconvex metric spaces we offer an exposition of progress in this article. Therefore, we cover primarily observations where the mapping requirements are metric. We shall recall effects of non-expansive proximinal retractions and their effect on the principle of best approximation and best pairs for proximity. Finally, few reflections and new findings are shown on the expansion of compact maps.

Keywords: hyperconvex metric spaces, proximinal retractions

I. INTRODUCTION

In mathematics and applied science, metrics are very important. Therefore several scholars sought many ways to generalise metric spaces. For instance, 2-metric space concepts and D-metric space concepts were introduced. The new system of widespread metric spaces called G-metric spaces has been implemented, to establish a new theory of fixed points for different mappings, as the generalisation of metric spaces (X,d) in this new structure. Introduced a D-metric spaces likely to change D-metric space description by adding some simple properties in the D-metric spaces, which have been shown.

S-metric space was newly added. The S-metric space is a 3-dimensional space. Extensive mapping studies are an interesting field of investigation of theory of fixed points. The study of vast mappings in fixed point theory is a very important field of inquiry. The principle of expanding cartoons was presented in 1984 and some fixed point theorems have been illustrated in absolute metric spaces. We refer the reader to more information on mapping expansion and related outcomes. We function in S-metric space in our article. Many of these results often use arbitrary mappings under various expansive forms of circumstances.

1.1 Definitions

Let IR denote the set of all real numbers, 7L denote the set of all integers and IN denote the set of all natural numbers. If / is a function from a set X to itself, we denote

$$f^2(x) = f(f(x))$$

We define PER(X) = {Per(f) : / is a continuous self-map of X}.

This is a family of subsets of IN

Sarkovskii's Theorem, PER (1R) and PER(I)

Let f be a continuous function from IR to IR. Then one can prove "If f has a point t with period / > 1 then f has a fixed point (i.e. point of period 1)". This provokes us to think about the following questions:

Theorem 1.1: If 3 € Per (f) for a continuous function f: IR -> IR, thcnPer(f) = IN

In other words, period 3 => period n for all n. [We say period rn =$> period n, if the existence of a point of period m implies that of another point of period n].

They also provided a counter example in that paper to demonstrate that period 5 does not mean period 3. Unknown to Li and York, Russian mathematician A.N. In 1964, Swarovski had already addressed the general question of when period rn means period n for continuous self-maps on a real graph.

He introduced a new total ordering on the natural numbers as follows:

Theorem 1.2: Suppose /: IR —> IR is continuous. Suppose / has a periodic point of period k. If k t> / in the above ordering, then / has a periodic point of period / also . From the theorem we make the following observations:

1. If per (f) is finite, then every element of Per (f) is a power of 2.

2. Period 3 is the greatest period in the order of the Sarkovskys, and thus period 3 indicates the presence of all other ages. As a consequence, the main result of this theorem becomes a corollary.

The converse of Sarkovskii7 s theorem is also true.

II. REVIEW OF LITERATURE

Abdul Latif (2012) -Therefore the general result is either strengthened by many known fixed point effects. First, we give a theorem that generalises the concept of Banach contraction and the set point theorems of many authors, then a fixed point theorem for a contraction that is multifunctional (0,l) and weak.

Ali Akbar and Binayak S. Choudhury (2012) -Recently some issues with fixed point theory have included control functions. In this article we introduce to given maps in quasi-metric spaces the definition of generalised weak contraction. In this paper, we get constructive fixed point theorems for individual operators in an implied relation defined general class of almost contractions.

SirousMoradi and ArezooBeiranvand (2010) In this paper we examine the existence of fixed points for complete metric mapping (X,d) that satisfy general contractive inequality based on a different mission. There are three mappings where T is generalised (f,g)-weak contractive mapping on an unempt part of the branch space. The existence of common fixed points is defined.

A.Beitollahi and P. Azhdari(2011) –The fuse-like mapping principle has been introduced and theorem for fuse-like contraction mappings has been proven. Two results on random fixed points are given in this paper. For random multi-value contraction mapping the first results are generalised. The second result is an altered version of nonlinear mapping results.

VasileBerinde and Francesca Vetro(2012)- The coincidence and common point theorems for self-mapping in the general class of contractions described by implicit relationships in this paper are obtained by the setting of matric or ordered metric spaces. In this paper a partial order for a non-archimedean flux metric space is given under the Lukasiewcz t-norm and fixed point mapping theorems are illustrated for single and multi-value mappings. Recently in 2011 a new definition was developed for the cone metric space c-distance. It seeks to expand and generalise some fixed c-distance theorems in cone metric space. The definition of a three-point fixed point in part-ordered metric spaces for single mappings was introduced.

Rajarajeswari and Dhanalakshmi (2014) broadened a few activities on it and characterized Interval esteemed intuitionistic fuzzy delicate grid, its sorts with models and their properties of certain administrators based on loads. Have broadened the strategy in utilizing intuitionistic fuzzy delicate set theory Connected fuzzy delicate set theory through surely understood Sanchez's methodology for restorative conclusion utilizing fuzzy number juggling tasks and display the strategy with a speculative contextual investigation connected with the thought of fuzzy delicate sets in Sanchez's technique for medicinal determination.

Manoj Bora et al (2014) made an endeavor to present the fundamental idea of intuitionistic fuzzy delicate grid theory. Further a similar idea of intuitionistic fuzzy delicate grid item has been connected to take care of an issue in therapeutic determination. The transportation issue has been one of the most punctual applications of direct programming issues. The fundamental transportation issue was initially created by Hitchcock (1978). Antiquated techniques for arrangement got from the simplex calculation were created in 1947, basically by Dantzig (1951) and after that by Charnes and Cooper (1954). The transportation issue can be displayed as a standard straight programming issue, which would then be able to be tackled by the straightforward technique.

III. DEFINITIONS OF FIXED POINT

We begin our studies with the following known definitions;

Definition 2.1:*Let (X,d) be a metric space and let* {p „} *be a sequence of points in X , then it is said to be a Cauchy sequence in X if and only if for every* e > 0 *there exists a positive integer n{e) such that, m , n > n(e) => d(pm,pn) < e.*

It is clear that every convergent sequence in a metric space is a Cauchy sequence but the converse need not be true. A metric space (*X*, *d)* is said to be complete if and only if every Cauchy sequence in *X* converges to a point in *X.*

Definition 2.2:*A self-mapping of a metric space {X,d) is said to be Lipschitzian*

*if for all x ,y € X and a >*0

d(f{x),f(y)) < a d(x,y).

T Is said to be contraction on a if a E[0,1) *and no expansive if a =* 1. *A contraction mapping is always continuous.*

In 1922, S. Banach's contraction principle appeared and this was known for its simple and elegant proof by using the Picard's iteration in a complete metric space. Banach's fixed point theorem states;

Theorem 2.1. *Let X be a complete metric space with metric d and f : X —>X is required to be a contraction, that is there must exists a* <1 *such that, the conclusion is that, f has a fixed point, in fact exactly one.*

$$d(f(x), f(y)) \leq \alpha\, d(x,y),\ \forall\, x,y\ \in\ X,$$

Proof: Let *x* 6 *X* be an arbitrary element. Starting from *x* we form the iterations,

$$x_1 = fx,\ x_2 = fx_1,\ x_3 = fx_2 \ldots\ldots,\ x_n = fx_{n-1} \ldots$$

We verify that *{x„}* is a Cauchy sequence. We have,

d(xi,Xi) = d (f x , f x i) < a d(x,xi) *= a d(x,fx)*

d(x2, x3) = d (f x i , f x 2) < a d(xi,x2) < a 2 d(x,fx)

d(x3, x4) = d(fx2, f x 3) < a d(x2, x3) < a 3 d(x,fx) .

In general, for any positive integer *n*, *d(xn,x n+*1) *<an d(x,fx) .* Also, for any positive integer *p,*

$$
\begin{aligned}
d(x_n, x_{n+p}) &\le d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots\cdots + d(x_{n+p-1}, x_{n+p}) \\
&\le \alpha^n\, d(x, fx) + \alpha^{n+1} d(x, fx) + \cdots\cdots + \alpha^{n+p-1}\, d(x, fx) \\
&= (\alpha^n + \alpha^{n+1} \cdots\cdots + \alpha^{n+p-1})\, d(x, fx) \\
&= \frac{\alpha^n - \alpha^{n+p}}{1-\alpha}\, d(x, fx) < \frac{\alpha^n}{1-\alpha}\, d(x, fx),\ (\text{since } 0 < \alpha < 1).
\end{aligned}
$$

Since *a*< 1, the above relation shows that *d(xn, xn+p)* —►0 as n —» oo. Therefore {x „} is a Cauchy sequence. Since *X* is complete, the sequence {x n} converge to a point *xo* (say) in *X*.Now we show that *f x 0 = xQ,* for this by triangle inequality we have,

$$
\begin{aligned}
d(x_0, fx_0) &\le d(x_0, x_n) + d(x_n, fx_0) \\
&= d(x_0, x_n) + d(fx_{n-1}, fx_0) \\
&\le d(x_0, x_n) + \alpha\, d(x_{n-1}, x_0) \to 0 \text{ as } n \to \infty.
\end{aligned}
$$

So *f x* o = Xo- Therefore xo is a fixed point of. Uniqueness can be easily check using contradiction method. So / has a unique fixed point in *X.* n 1844. Proved the existence and uniqueness of the solution of the differential equation ^ = *T(x,y)\ y(x*o) = *yo,* where *T* is a continuously differentiable function. Later in 1877, simplified Cauchy's proof, in 1890 attempted a deeper result of Cauchy's theorem by supposing only the continuity of *T.*

Definition 2.11*: A set X is said to have a fixed point property (FPP) if each continuous mapping f : X —* X of this set into itself has a fixed point.*

In, it is appeared fixed point property is a topological property. Truly the investigation of fixed point theory started in 1912 with a theorem given by famous Dutch mathematician (1881-1966). This is the most famous and significant theorem on the topological fixed point property. It tends to be detailed as; the shut unit *ballBn* € R" *has the topological fixed point property.* Its disclosure has had a gigantic impact in the improvement of a few parts of mathematics, particularly logarithmic topology.

A significant generalization of theorem was found in 1930 it might be expressed as pursues: Any non vacant, conservative arched subset K of a Banach space has the topological fixed point property.

The theorem for a compact map, known as the second form of the above mentioned theorem, also proved in 1930. We need the following description before stating the theorem.

IV. METRIC SPACE

In this section we provide Metric Space definitions and some other relevant authors' definitions and findings that we refer to in the following chapters.

Include the definition of the metric abstract space, in which the notion of the distance occurs, which offers a popular idealisation of various scientific, physical and mathematical systems. The subject matter can be most different. They may be points, features, sets, and even subjective sensational experiences. The probability that a non-negative real number is associated with each ordered pair of elements of a certain set is essential and that the numbers associated with those elements fulfil those conditions.

$$(i):\ d(p,q) = 0 \text{ if and only if } p = q \quad (\text{Identity})$$

$$(ii):\ d(p,q) \geq 0 \quad (\text{Positivity})$$

$$(iii):\ d(p,q) = d(q,p) \quad (\text{Symmetry})$$

$$(iv):\ d(p,r) \leq d(p,q) + d(q,r) \quad (\text{Triangle Inequality})\text{ ”}$$

In 1922: Proveed a theorem that guarantees the existence and uniqueness of a fixed point under suitable conditions.

In 1961: Proved a fixed point theorem on Contractive mappings.

Contractive mapping is clearly continuous, and if such mapping has a fixed point, this fixed point is clearly unique. Denote by admixing the family of the actual functions (submitted) to (submitted) to (submitted) to (submitted to) the following conditions:

$$(i): \alpha(x,y) = \alpha(d(x,y)),\ i.e.,\ \alpha \text{ is dependent on the distance between } x \text{ and } y \text{ only}$$

$$(ii):\ 0 \leq \alpha(t) < 1 \text{ for every } t > 0$$

$$(iii):\ \alpha(t) \text{ is a monotonically decreasing function of } t.\text{ ”}$$

Theorem 1.1.3. “Let ؈ be a contractive mapping of a complete metric space ॆ into itself such that there exists a subset ⊃ؾ and a point ؿ ∋ □ satisfying the following:

$$(i): d(x,x_0) - d(Ax, A\,x_0) \geq 2\, d(x_0, A\,x_0) \text{ for every } x \in X/M$$

$$(ii): d(Ax,Ay) \leq \alpha(x,y) d(x,y) \text{ for every } x \in X, \text{ where } \alpha(x,y) = \alpha\big(d(x,y)\big) \in F_1.$$

Then there exists a unique fixed point. ”

$$H_\nu(P,Q,t) = \max\{\sup_{p\in P} \nu(p,Q,t), \sup_{q\in Q} \nu(q,P,t)\},$$

for all $P, Q \in K(U)$ and $t > 0$.

V. CONCLUSION

The theory of fuzzy setting and the fluid relationship is ideal mathematical tools to set fluid metric spaces and fluid metric graph spaces. A fuzzy set and fluffy relationship in fluffy topological space can be adopted to formally describe flow metrical spaces and flouted diagram metric space. There are various topical qualities.

The relationship between fixed point properties and fuzzy metric spaces was generally studied in this research. It should be remembered that, using various mappings, the analysis is only used to create new fixed-point theorems.

In short, the thesis presents a theoretical context in which theorems are modelled. More research on the representation of fuzzy metric graphic spaces should be made. The thesis generally deals with certain methods in different approaches for generating fixed point theorems.

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