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# Splitting Messages To Send From Source To Destination In The Minimum Amount Of Time In A Flow Network

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## Abstract:

In a flow network, there is no restriction on the selection of paths on which data is sent. Only the edge capacities are important, we generally search for the shortest path with high bandwidth to maximize the transmission of data, but this paper suggests that the shortest path is not that important if it has low bandwidth. For quick data transmission, it would be better to split a given message (data) among several paths in a flow network, The distribution of a message to different paths again depends on the transit time of the path and its capacity.

**Keywords:** Flow network, Transit time, Shortest path, Bandwidth.

## 1.0 INTRODUCTION-

An nonsplittable flow problem is one in which flow is routed along a single path from source S to destination D. A splittable flow problem is a general form of an unsplittable flow problem, which is introduced by Baier, Kohler, and Skuttlar [1, 4, and 10], To send the data from source S to destination D at most k paths can be used, for each kind of data there is an upper limit on the number of paths that may be used to route its demand.

An important property of network flow which occurs in the physical world is the variation in flow with respect to time, flow does not move instantaneously through a network but it needs a certain amount of time which is known as transit time to pass through each edge. The fastest Source S-Destination D flow problem is to send a given amount of flow from S to D in such a way that the last unit of flow reaches at the destination as early as possible. Ford and Fulkerson presented an algorithm flow over time [8] for a given a network with a single source node S, a single destination node D, and a time constraint. They consider the problem to maximize the flow per unit of time from S to D.[6, 14, and 16]

It may be assumed that transit time remains same throughout, so the flow in an edge moves with a uniform speed i.e. the transit times are constant. For edge  $e$  the transit time  $\sigma(e) \in \mathbb{R}_+$  is independent of the capacity and the flow value in the edge.

A restriction is imposed on the length of the selected path, all the S-D paths are disjointed and their length will be bounded above by the bound  $L_m$ . [3, 4, and 5]

The  $k$ , S-D paths denoted by set  $P = \{P_1, P_2, \dots, P_k\}$  all the paths are  $L_m$  length bounded for some  $L_m \in \mathbb{R}_+$ . [12 and 13].

$$\text{So } \sum_{e \in P_i} l(e) \leq L_m$$

For Un splittable flow: Let  $M$  is the message length which we want to send from source  $S$  to destination  $D$ , since flow is un splittable so a single path  $P([v_1, v_2, \dots, v_p])$  is used in the flow network  $N(G)$ . [7, 8 and 9]

The transit time along path  $P([v_1, v_2, \dots, v_p])$  is defined as the sum of the link transit time i.e.

$$\begin{aligned} \text{Transit time } t \text{ on the path } P([v_1, v_2, \dots, v_p]) \text{ is} \\ = \sigma([v_1, v_2, \dots, v_p]) = \sum_{i=1, 2, \dots, p-1} \sigma(v_i, v_{i+1}) = \sum_{e_i \in P_i} \sigma(e_i) \end{aligned} \quad \dots (1)$$

The smallest path from node  $u$  to node  $v$  in the flow network is the path which has the minimum transit time of all paths from  $u$  to  $v$  in the flow network. [6 and 7]

A path capacity (bandwidth) is defined as

$$C(P) = C([v_1, v_2, \dots, v_p]) = \min_{1 \leq i \leq p-1} C(v_i, v_{i+1}) \quad \dots (2)$$

Also, the message in path  $P$  transmit with flow value  $F(P)$  where

$$F(P) \leq C(P) = C([v_1, v_2, \dots, v_p])$$

There is no intermediate storage so the flow in is equal to flow out at all intermediate nodes.

If the front end of the message is transmitted from  $v_1$  at time  $t=0$  along the path  $P([v_1, v_2, \dots, v_p])$  then the endpoint of the message would arrive at  $v_p$  at time

$$T(\text{reaching} + \text{receiving}) = \sigma([v_1, v_2, \dots, v_p]) + \frac{M}{F(P)} = \left\{ \sum_{e_i \in P_i} \sigma(e_i) + \frac{M}{F(P)} \right\} \quad \dots (3)$$

It is the receiving time of the message, where  $M$  is the message length.

The quickest path from a node  $v_i$  to  $v_j$  in a network for a message of length  $M$  is the path between  $v_i$  to  $v_j$  that gives a minimum end-to-end delay time for the message among all paths from  $v_i$  to  $v_j$  [7, 8, and 15].

## 2.0 Splitting of message for fast transmission-

Consider the flow network  $G(V,E)$  with sources node  $S$  and a destination node  $D$  and with a message  $M$ , the capacities of edges and the transit time for each edge of the network are given. The objective is to transfer the message of length  $M$  from source  $S$  to the destination  $D$  in a minimum amount of time for this we will split the message  $M$  and assign the components to various paths from source  $S$  to destination  $D$ . [2, 10 and 11]

To minimize the time to transmit the message length  $M$  to the destination  $D$  from source  $S$ , we will use strategies,

- i) maximize the transmission rate for each path,
- ii) selection of the shortest path, and the distribution of the message segment to different paths
- iii) then the distribution of message  $M$  to different paths.

From the collection of  $(S-D)$  edge disjoint path  $P(S-D)$  (says) select those paths which are bounded by  $L_m$  and denote this set by  $P_{L_m}$ .

$P_{L_m}$  is the subset of set  $P(S-D)$ . Also  $|P_{L_m}| = k$

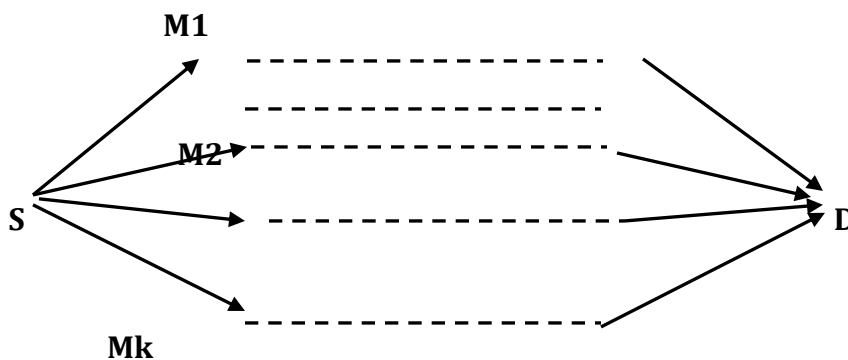


Figure -1.0 - splitting message in  $k$  -edge disjoint paths

The transit time along path  $P_i$  is

$$\sigma(P_i) = \sum_{e_i \in P_i} \sigma(e_i)$$

Sum of all flow along path  $P_i$  must be less than  $C(e_i) \forall e_i \in P_i$

$$\sum_{e_i \in P_i} F(P_i) \leq C(e_i) \forall e_i \in P_i$$

Thus transmission time and receiving time along path  $P_i$  is,  $T_i = \sigma(P_i) + \frac{M_i}{F(P_i)}$

The total flow in an Edge  $e_i$  cannot be higher than  $C(e_i)$  simultaneously sending message  $M_i$  with flow  $F(P_i)$  S to D along each path  $P_i$ . So that full message  $M$  reaches the destination in a minimum amount of time.

No storage at intermediate node i.e. flow in = flow out at all intermediate nodes.

$$M = M_1 + M_2 + \dots + M_k$$

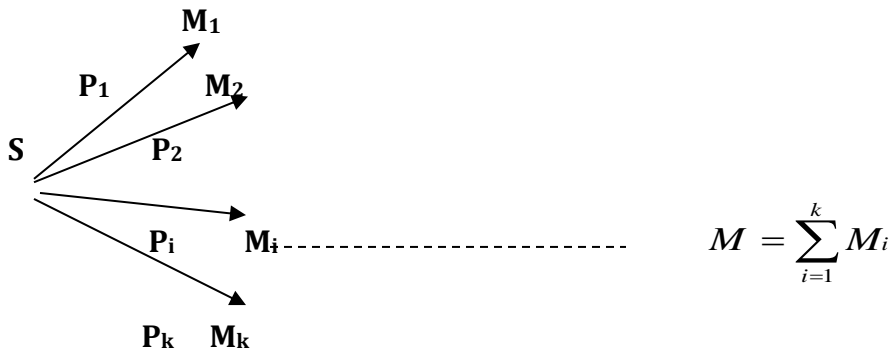


Figure-2.0 -Allocation of message component of the different paths.

### 2.1 Allocation of message component of the different paths:

It depends on the capacity of the path and the transit time along the path, the low transit time and high capacity paths will be preferred

$$T_i = \sigma(P_i) + \frac{M_i}{F(P_i)}$$

Reaching time                      receiving time

Maximum time

$$T_{\min} = \max_{1 \leq i \leq k} \left\{ \sigma(P_i) + \frac{M_i}{F(P_i)} \right\} \dots (4)$$

Flow is simultaneously and the message segment distribution to different paths is on the basis of path capacity and transit time

$$M_i \propto \frac{C(P_i)}{\sigma(P_i)} \dots (5)$$

$$M_i = D \cdot \frac{C(P_i)}{\sigma(P_i)}$$

Where D is the constant of proportionality

### 3.0:Numerical illustration:

Given a flow network with edge capacity and transit time.

$M(t)$  is the maximum size of the message that can be transmitted from source S to destination D in time  $t, t=0,1,2, \dots$ ,

Here we can express  $M(t)$  as a non-decreasing linear function of time  $t$ .

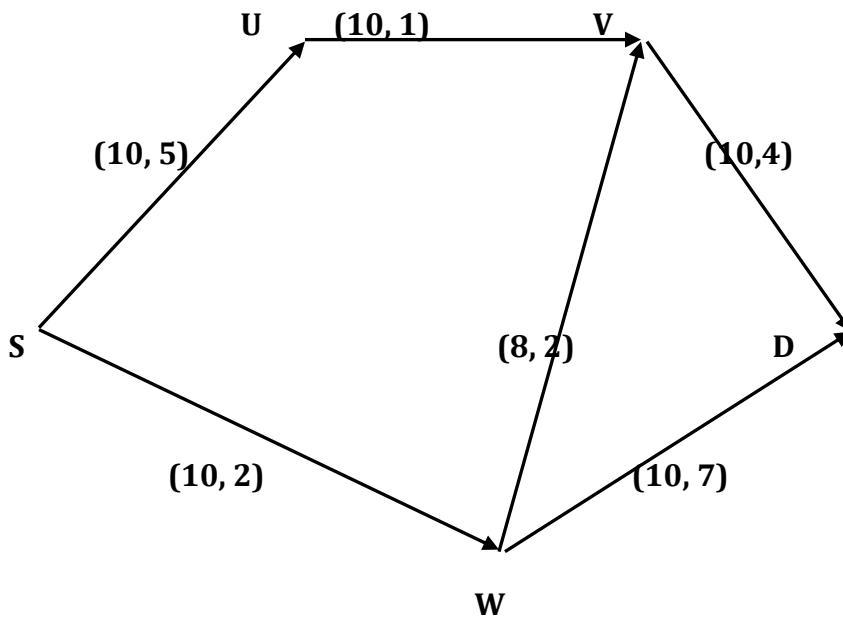


Figure -3.0 Flow network with edge capacity and transit time

In the figure path (S-W-V-D) is the smallest route from S to D with transit time 8 time unit.

The max amount of flow which can be routed along path (S-W-V-D) is =  $\text{Min} \{ \text{residual capacity of edges (S-W) (W-V) and (V-D)} \} = \text{Min} \{ 10, 8, 10 \} = 8$ .

Similarly we can calculate for other edges.

IF  $M(t)$  is the maximum amount of Message-(data) (Max flow amount along paths)

$$0, \quad 0 \leq t < 8$$

$(8t-64), 8 \leq t \leq 9$  , (S-W-V-D),(8)

$M(t) = (10t-82), 9 \leq T \leq 10$  , (S-W-V-D) (S-W-D),(8,2)

$(12T-102), 10 \leq T \leq 11$  , (S-W-V-D), (S-W-D), (S-U-V-D),(8, 2, 2)

$(20T-190)12 \leq t$  , (S-W-D)(S-U-V-D),(10, 10)

The maximum sum of transmission rate along the paths from S to D is  $10+10=20$ .

Suppose we want to send a message of length 112 from source S to destination D, there are three paths available, using the scheme of distribution of message and dividing the message into three components 36, 40, and 36 based on capacity and transit time, and send these components along the path (S-U-V-D), (S-W-D) and (S-W-V-D) respectively at the flow rate 10, 10, and 8 unit per unit of time, the entire message will be received in 13.6 unit of time at destination D.

Again if we use the shortest single path (S-W-V-D) the transmission time would be quite lengthy and we will not get the optimal result.

#### 4.0 CONCLUSION-

The work in this paper suggests that the quick transmission from source S to destination D, can not always be achieved by using the shortest paths or along a single path, but it depends on the magnitude of the message which we want to transmit. The best strategy of distributing the message to different edge disjointed paths according to the size of the message leads to the fast transmission of the message and optimizes resource utilization. The most important point of path selection is the transit time and the bandwidth of the paths; we will prefer those paths which are having high bandwidth and less transit time.

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