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Pre-service teachers' explanations of primitive terms: point, line and plane

Mustafa Akıncı, *Zonguldak Bülent Ecevit University*, Turkey, *mustafa.akinci@beun.edu.tr*
ORCID: 0000-0003-2096-7617
Murat Genç, *Zonguldak Bülent Ecevit University*, Turkey, *muratgenc@beun.edu.tr*
ORCID: 0000-0003-4525-7507

Abstract. The purpose of this study was to investigate how pre-service elementary mathematics teachers explain three primitive terms: point, line, and plane. A qualitative exploratory case study design was conducted with 61 participants. The data collected by an open-ended questionnaire was analyzed through the content analysis technique by utilizing NVivo software. The findings showed that pre-service mathematics teachers tended to use signs related to both characteristics and the essence of the terms. This can be due to the fact that terms tried to be explained are the primitive terms and that the explanations given by Euclid about these terms are not easy to understand. In some explanations, more than one sign was used to make the definition more understandable. However, although the signs used for explaining the term seem to constitute its definition, these signs cannot always literally represent this term and cannot be used in place of that term.

Keywords: Euclidean geometry, primitive terms, definition, point, line, plane, sign

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INTRODUCTION

Definitions have an important function in communicating the meanings of a concept and in providing unity on meaning (Shir & Zaslavsky, 2001). It is noted that not every expression that describes or lists the features of a concept in mathematics can be a mathematical definition but they are merely the descriptions that describe such concept (De Villiers, 1998). There are certain principles while making a mathematical definition, however, from an educational point of view, the definitions made in compliance with such principles may be insufficient for the students to make sense of such definition (Poincaré, 1909/2009). Since the definitions being the building blocks of mathematical thinking have an important role in explaining a mathematical concept and distinguishing one concept from others, it is claimed that mentioning the properties of a concept while describing it facilitates making sense of such concept (Çakıroğlu, 2013). Therefore, the properties and descriptions of respective concept while the prospective teachers are describing mathematical concepts are crucial in order for make sense of such concepts.

When the studies on the primitive terms of Euclidian geometry are observed, it is seen that such studies are aimed at determining the perception, representation and misconceptions of the prospective teachers and primary educations students (Dane & Başkurt, 2012; Dane, 2008; Kılıç, Temel, & Şenol, 2015; Kiriş, 2008; Öksüz, 2010; Şengül & Dereli, 2009; Tuluk, 2014; Yenilmez & Yaşa, 2008). For example, in study conducted with primary education students, Kiriş (2008) attempted to determine the misconceptions regarding the concepts of point, line, line segment, half-line and plane of sixth grade students as well as the ways of thinking that lead to such misconceptions. In conclusion, the misconceptions found are contained under main topics intended for associating geometrical concepts with daily life, using the properties of such concepts in operational questions and linking geometrical concepts together. Similarly, in another study conducted by Dane and Başkurt (2012), the perception level and misconceptions of 6th, 7th and 8th grade students regarding undefined concepts of geometry are aimed to be determined. Opinions of students on the concepts of point, line and plain are collected by means of three semi-structured open end questions and as a result of the analyses it is established that the students have difficulties in interpreting the concepts of point, line and plain and also have

various misconceptions. In another similar study, Öksüz (2010) aimed to reveal the difficulties and misconceptions being faced by the gifted students regarding the concepts of point, line, line segment, ray and plain. As a result of the study, it is found out that the students have many difficulties in conceptualizing such subjects and also have various misconceptions.

When the studies conducted with prospective teachers are examined, Tuluk's (2014) study comes to fore due to its objective of determining the representations used by prospective teachers studying in the first grade program for point, straight line, surface and space concepts and of interpreting such representations in terms of area knowledge and area teaching knowledge. In this study, which contains 9 open-ended questions to be asked to subjects as a data collection tool, representations in different categories of related concepts were obtained. In another study, Kılıç et al. (2015) used the Geometry Basic Concepts Test (GTKT) that they developed as a data collection tool, and presented the misconceptions assumed by senior prospective mathematics teachers who received formation training in the same university, regarding the concepts of point, line, plain and angle. As a result of the research, it was observed that the prospective teachers have the misconception about the dimensions of the concepts of point, line, line segment, ray, half-line, plane and angle. In this context, it has been observed that the subjects have misconceptions that the concept of plane consists of closed geometric shapes, the concepts of line and line model are confused with each other, and the concept of line is measurable and limited. In this context, unlike other studies, the objective in study was exploring the different descriptions of prospective elementary mathematics teachers based on the qualities they use in defining the concepts of point, line and plane, which are the primitive terms of Euclidean geometry.

Purpose and Significance of the Study

Considering the importance of the definitions being the basic building block of mathematical thinking and which allow us to distinguish a concept from others, it is necessary for teachers to think flexible enough while defining mathematical concepts and to evaluate different definitions according to their pedagogical characteristics (Leikin & Winicki – Landman, 2000). On the other hand, along with the importance of representation awareness in mathematics education, it is advocated that the ability to choose a representation in line with the relationships given for prospective teachers and to create appropriate representation for such relationships will make important contributions to the conceptual structuring of knowledge on conceptual level (Kaput, 1998). In the face of this situation, the individuals with representation awareness are expected to have stronger schemes regarding the relevant concept and thus use the most appropriate representation for the problem. In other words, the expressions used by the individuals in the process of defining mathematical concepts are crucial in order to make sense of the concepts. In this context, it is important for mathematics education researchers to examine how prospective teachers visualize mathematical concepts and explain them in their minds. Therefore, the objective of this study is to examine different descriptions of prospective elementary mathematics teachers on these concepts based on the signs they use while explaining the concepts of point, line and plane.

Conceptual Framework

The science of mathematics consists of theorems and definitions derived from a set of undefined terms and axioms (Clark, 2012). The terms that are not defined in mathematics are referred to as primitive terms (Thompson, 2007). Euclid, in his book Elements, has tried to explain these terms, which are expressed as undefined, and suggested that the point is that which has no part, the line is breadthless length, and the plane is that which has length and breadth only (Fitzpatrick, 2008).

When considering the dimension of teaching undefined terms, it is seen that formalist and intuitive approaches come to the fore. The formalist approach prefers to express primitive terms with symbols by transforming them into symbolic axiomatic structures rather than to explain them by reducing them to logic (Hilbert, 1899). Similarly, the Ministry of National Education [MEB] (2018) emphasizes displaying and naming of dots and lines by means of symbols in

primary education mathematics curriculum. On the other hand, according to the intuitive approach, mathematics consists of a mind building (Baki, 2014). Within this frame, in order for the primitive terms to be imprinted on one's mined, it is expressed that a point is the mark left by a pointed pen on the paper, a line is a line fragment of which both ends extend to infinity in a straight manner without being broken, and the plane is a surface extending to all directions infinitely (Mahir, 1999; School Mathematics Study Group, 1961). However, in order to avoid such descriptions to be perceived as definitions, it was emphasized that none of these expressions are definitions but merely reflections of the ideas that the terms evoke in our minds. Similarly, Altun (2005a, 2005b) suggested for the undefined concepts to be adopted by the students by means of making such concepts understood through activities. He stated that the representations of the point in primary education can be expressed as a trace left by the pencil on paper, a small grain of sand or granulated sugar, and that the point can be explained with such examples. In the teaching of the line, he points out that the geometric object formed by the points that we often put with the help of a ruler constitute the line, so that the line can be described as a set of points, and it must be noted that the line extends to infinity at both ends. In the teaching of the plane, it is emphasized that the students' attention may be drawn to different surfaces such as table surface, paper surface, glass surface, surface of the blackboard or still water surface, and the fact that such surfaces extend infinitely to all directions.

There is also an approach that sees the point as the basis of geometric objects because it is abstract and has no width and dimension and tries to define such objects. For example, the point is seen as the building block of geometry rather than being defined, whereas the terms of line and plane are explained by the help of point (Altun, 2005a; Hızarcı et al., 2009). In this context, the line is expressed as a one-dimensional set of points without width and height extending to both directions infinitely (MEB Commission, 2017).

On the other hand, one of the most important aspects in the identification process is to define a new fact or object by means of the terms known to everyone. The definition can be seen as a concept used by the speaker in order to be understood better by the listener or the clarification of the meaning of a word, as well as the descriptions made with the basic features of the concept and taking its distinctive features into account. The factors to be considered while creating a mathematical definition have been discussed by various researchers and are the criteria that a definition should have are expressed as being suitable for hierarchical structure, describing an existing fact, being suitable for axiomatic structure and being economical (Van Dormolen & Zaslavsky, 2003; Winicki-Landman & Leikin, 2000; Vinner, 1991; Zazkis & Leikin, 2008). On the other hand, according to classical logicians, the definition is made in two ways, namely the definition related to the essence and related to the relation that emphasizes the quality of the term being studied on. The one related to the essence is a rather correct definition and it clearly indicates the place of concept and completely distinguishes it from others, while the one related to the relation gives some information about the quality of something. Accordingly, the definition is specific to the object defined and to the person who makes it (Öner, 1986). Since people's perceptions and interpretations of a concept or a fact may be different, their visualization and outpouring of the relevant concept or fact may differ in terms of the descriptions they make and of the signs such as the graphics, pictures, equations, symbols, virtual objects, etc. they use (Duval, 2006). Apart from these, there are studies suggesting that the gestures used during mathematical conversations have meanings and also have functions in communicating, and therefore evaluating gestures as signs (Akıncı & Arıkan 2017; Arzarello, Paola, Robutti, & Sabena, 2009; Edwards 2009; Radford 2009).

Within this frame, it will be useful to consider semiotics, which examines the signs that accompany this process, during the process of explaining a mathematical concept and ideas. Semiotics is a sign science that examines how meaning is formed rather than what meaning is. When we look at the history of semiotics, it is seen that an English philosopher John Locke was the first person to use the term semiotics teaching in his works. According to J. Locke, the objective of semiotics teaching is examining the quality of signs used by the mind to understand things or to make them understood by others (Rıfat, 1992). Saussure, one of the founders of contemporary semiotics, has explained the term of sign as something represented by another

thing that replaces it. Saussure (2001) suggested that the sign consists of two parts being the indicative and the indicated. Vardar et al. (1988) mentioned the sign as any kind of object, entity or fact that is capable of replacing something else and indicating something other than itself. Signs not only allow us to think more effectively on the problem, but also enable us to transfer our thoughts on paper and communicate with others easily (Kaput, 1995). We make use of signs to understand and explain mathematics. Although signs are a way of expressing abstract objects more easily, such signs must never be confused with mathematical objects themselves (Saenz-Ludlow & Presmeg, 2006). Therefore, it is important to tell the object and its representative apart, and to know that no representation of a mathematical object can reflect it exactly (Duval 2006). It has been suggested that the observable representation systems should be used not to embody mathematical objects but to be able to comment on such objects (Duval, 1999).

Furthermore, three different signs were mentioned in the Semiotic Mediation Theory developed by Bartolini Bussi and Mariotti (2008). These signs entered into Turkish literature by means of a study by Turgut (2017) in the form of artifact signs, pivot signs and mathematical signs. Artifact signs are signs based on students' perceptions and observations that do not contain mathematical thoughts, which they create as a result of their interaction with the artifact. Pivot signs are very meaningful non-mathematical signs that accompany the transition of students from their own thoughts to mathematical thoughts. Mathematical signs, on the other hand, are the signs that point at the formation of mathematical ideas such as the definitions, expressions, and claims that are accepted by the students' mathematics culture and constitute the target of semiotics mediation process being organized by the teacher.

METHOD

Research Design

In this study, the case study, being one of the qualitative research methods, was used to explain how the prospective teachers express primitive terms. Creswell (2009) expresses qualitative research as a process of interpretation by questioning with specific methods for an event to be understood in a comprehensive and detailed manner. Akar (2016) suggests that the case study requires an in-depth questioning of how individuals see themselves, their perceptions and emotions according to the context, and the underlying causes. Therefore, the exploratory case study has been determined as the most appropriate research design to understand the expressions used by prospective teachers for explaining the concepts of point, line and plane, being the primitive terms (Yin, 2014).

Participants

This study was conducted at the end of the spring semester with a total of 61 junior and senior prospective teachers who have taken Special Education Methods, Analytical Geometry and Geometry courses in the primary education mathematics teaching program of a state university. Therefore, the criterion sampling method, being a purposive sampling method, was used in determining the participants (Yıldırım & Şimsek, 2016).

Data Collection Tool

In this study, the data were obtained by means of a questionnaire consisting of three semi-structured open-ended questions in order to determine how prospective teachers can express the concepts of point, line and plane while explaining such concepts in the classroom environment in which they will teach. For the clarity of the questions in the questionnaire, the opinions of three prospective teachers and one field specialist were obtained and the questionnaire was validated. Accordingly, the first question was “How would you explain the concept of point in classroom environment?”, the second question was “How would you explain the concept of line in classroom environment?” And the third question was, "How would you explain the concept of plane in classroom environment?” A questionnaire containing these questions was applied to prospective teachers and the researcher was together with the participants during the application in order to answer their questions about the questionnaire

when necessary. By means of the open-ended questionnaire, the researcher and the subject were provided with time flexibility and also more systematic and comparable information was obtained from prospective teachers. During the questionnaire application, the objective of research was explained to the prospective teachers and it was stated that providing answers to the questions in the questionnaire in a sincere and objective manner is important for the research to achieve its objective.

Data Analysis

In this study, the data analysis was carried out by using content analysis method, which requires the in-depth analysis of collected data and allows to reveal previously unknown themes and relationships (Corbin & Strauss, 2015). In the research, firstly, the prospective teachers were coded within the range of OA1-OA61 and their answers to open-ended questions were examined by the researchers line by line, in detail and systematically, and a code list was created by assigning a keyword for each meaningful sentence of prospective teachers. After the coding phase was completed, similar codes were combined under meaningful categories in line with the objective of research. For example, the category of 'Non-dimensional or volumeless' for the concept of point was created by combining the codes obtained from the descriptions made by the prospective teachers such as "*Structure without dimension*", "*Is that with no width, direction*", "*Is that not occupying place in space*", "*Object without a certain height, dimension*" and "*Dimensionless structure*". The category of 'direction or linearity' for the concept of line was created by combining the codes obtained from the descriptions made by the prospective teachers such as "linear points", "meeting in the same direction", "is that with direction, orientation, length", "is that being linear", "continuous dots in the same direction and orientation" and "adjacent linear points". The category of 'Region, area, shape or surface' for the concept of plane was created by combining the codes obtained from the descriptions made by the prospective teachers such as "*Surface of paper*", "*Surface of table*", "*Top of the table*", "*Surface of board*", "*The region formed by two intersecting lines in space*", "*The infinite region between two intersecting lines*" and "*The shape formed by three non-linear points*". In this study, the categories created are expressed as keywords trying to explain the related term. Some descriptions regarding the respective concepts made by the participants were analyzed with one keyword, while some others were analyzed with more than one keyword. For example, the expression "*A trace without width and length*" that a participant made for the point is included in both the 'Trace' and 'Non-dimensional or volumeless' categories. Therefore, two keywords have been assigned to this description. Thus, the keywords used by the participants in order to make these concepts understandable were put forward. These keywords were first examined in the form of an essential description or a qualitative description that puts forward the feature of the respective term. On the other hand, since these keywords are the signs that point at the concepts of point, line and plane, the classification of these signs is made within the framework of semiotics mediation theory.

Frequencies of the keywords were calculated by taking into consideration the frequency of the repetition of keywords by the participants. After the frequency calculations, the ones among the participants that reflect their views in the most striking way are directly mentioned. During all these stages, what was searched for in the content of data was constantly questioned by considering the objective of research and allowing the opinions given place under the same category to form a meaningful whole was regarded (Patton, 2002). Qualitative data analysis software NVivo11 is used for data coding and classification during the creation of categories in the study due to its effective non-digital and non-processed data configuration features and the acquired findings are presented as figures and graphics. While presenting the findings to ensure the validity of research, citations containing the opinions of participants were frequently given place and the results were interpreted based on such opinions. At the same time, the main results produced as a result of the research were shared with the individuals participating in the research and their opinions were received. The purpose herein is to describe the situation correctly and objectively so that the produced concepts can form a consistent and meaningful whole within themselves (Merriam, 2013). In order to ensure the reliability of the research, the

researchers first determined the consensus and disagreement over the categories by coding the opinions written on the questionnaire forms by the prospective teachers separately, and then the reliability of research was calculated as 0,92 (Miles & Huberman, 1994). In addition, in order to further intensify the accuracy of findings and to enable the research results to reflect the opinions of participants accurately, the codes and categories created by a specialist in this field were checked and the consensus is reached accordingly.

FINDINGS

The descriptions provided by prospective teachers for primitive terms are discussed under the sub-headings of description of point concept, description of line concept and description of plane concept.

Description of a Point Concept

The answers given by the prospective teachers to the question "Can you describe the point?" were examined and the keywords used to explain the the term point in these answers have been determined as 'Trace', 'Non-dimensional or volumeless', 'Undefined', 'Basic or smallest unit', 'Indicating location or coordinate' and 'Intersection of two lines'. While each participant is indicated on the horizontal axis of the graph given in Figure 1, the keywords used by each participant to describe the concept of point are indicated on the vertical axis.

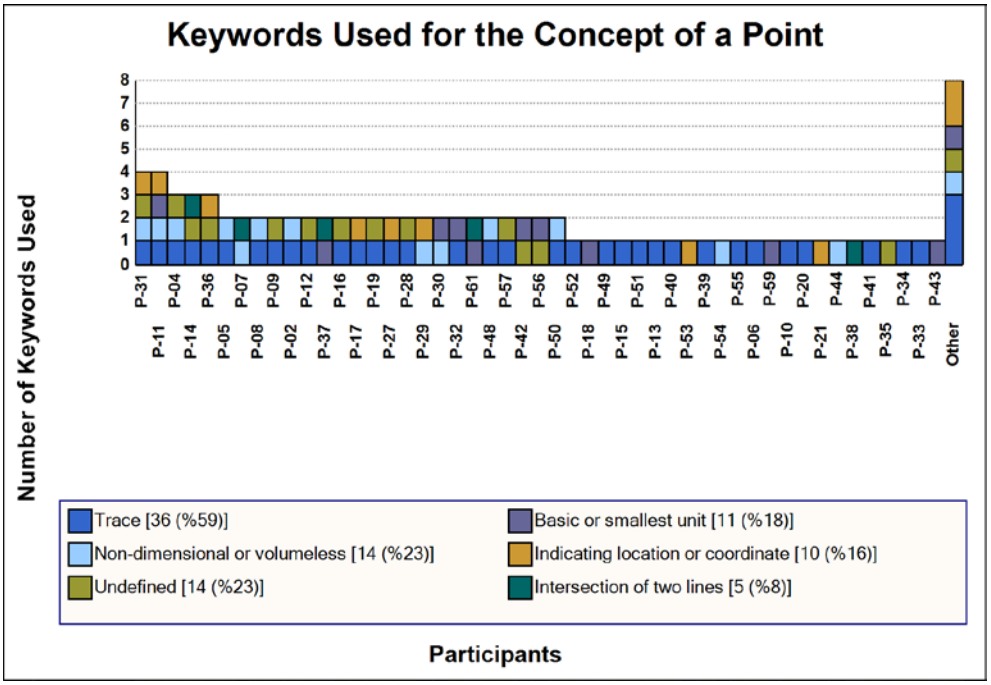

FIGURE 1. *Keywords used by prospective teachers to describe the concept of a point*

When the keywords used by prospective teachers to explain the point are examined in Figure 1, it is seen that the keywords 'Undefined' and 'Non-dimensional or volumeless' that belong to the essence of the point concept accompany the descriptions. It is observed that the prospective teachers who do not make any descriptions regarding the essence of the point concept are focusing on the physical appearance or correlations of the point concept and making use of the keywords such as 'Trace', 'Indicating location or coordinate', 'Basic or smallest unit' and 'Intersection of two lines'. When the descriptions about the point concept are analyzed within the semiotics mediation theory framework, it is seen that the descriptions made by the prospective teachers for the point such as 'Trace' and 'Basic or smallest unit' were pointing at pivot signs while the descriptions such as 'Non-dimensional or volumeless', 'Indicating location

or coordinate', 'The intersection of two lines' and 'Undefined' were pointing at mathematical signs.

It is established that a total of six signs were used 90 times in total by 61 prospective teachers. This showed us that some prospective teachers used more than one keyword while explaining the concept of point. Therefore, as seen in Figure 1, the use of keywords is sometimes done with a single word in the descriptions made for the point by the prospective teachers, while the use of keywords increased up to four, some other descriptions. The keywords accompanying the description of point are given under the following sub-headings.

Description of a point with one keyword

31 (51%) prospective teachers used one keyword to describe the point. 17 of these prospective teachers used the keyword 'Trace'. The description made by one of these prospective teachers is "*The mark left when a pen is touched on paper or on a board is called a point*" (OA10). On the other hand, 4 of 31 prospective teachers used the keyword 'Indicating location or coordinate'. The point description made by one of these prospective teachers was "*The representation of two ordered pair such as (x, y) on the coordinate plane*" (OA23) while the description made by another one was "*A concept indicating a location*"(OA53). Similarly, 4 of the 31 prospective teachers explained the point concept, this time, by using the keyword 'Basic or smallest unit'. The description made by one of these prospective teachers was "*The smallest building block of all objects. An equivalent of atom in chemistry*"(OA59), while the description made by another prospective teacher was "*It is the smallest unit that can be created on the plane*" (OA43). Other than that, 3 of 31 prospective teachers described the point by using the keyword "Non-dimensional or volumeless". The two descriptions in which this keyword is used are "*The object without a certain height, dimension*" (OA54) and "*Structure without dimension*" (OA44). Moreover, 2 of 31 prospective teachers described the point concept by using the keyword 'Undefined' and one of them by using the keyword 'Intersection of two lines'. An example description using the keyword *Undefined* is "*Undefined*. *I would say that there is no certain definition*"(OA35), while the description of the keyword 'Intersection of two lines' is "*The point where two lines intersect*" (OA38).

Description of a point with two keywords

The analysis of the keywords in the expressions made by 21 (34%) prospective teachers describing the point concept with two keywords are tried to be represented in Figure 2a, Figure 2b, Figure 2c and Figure 2d below.

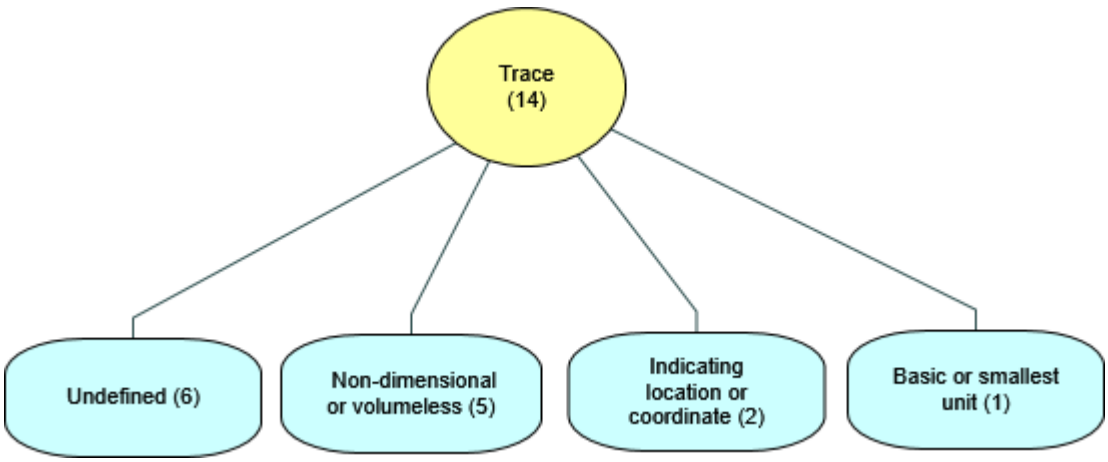

FIGURE 2a. *Description of a point with two keywords*

When Figure 2a is analyzed, it is seen that 6 of 14 prospective teachers used the keyword 'Undefined' together with the keyword 'Trace'. The descriptions made by two prospective teachers by using these two keywords are; "*The point is undefined. It is described as a trace left on*

the board by chalk and on the paper by pencil" (OA09) and "*It is the trace left by a pencil on the notebook as a figure. However, it is a geometric concept without definitionhaving an infinite greatness*" (OA12). It was observed that five prospective teachers used the keyword 'Non-dimensional or volumeless' together with the keyword 'Trace'. One of these descriptions was expressed as "*The trace of the pencil, which has no width length, left on the paper*" (OA05). Another keyword used with the keyword 'Trace' was the keyword 'Indicating location or coordinate'. This keyword was used with the keyword 'Trace' by two prospective teachers. One of these descriptions is "*The trace left on the notebook by the pen. It is any expression corresponding to (x, y) that indicates the location on a coordinate plane*" (OA17). It was observed that the keyword 'basic or smallest unit' was used by a prospective teacher together with the keyword 'Trace' and this description is "*The trace that occurs when I touch the pen on a plane and lift it back. It is the building block of all geometric shapes*" (OA32).

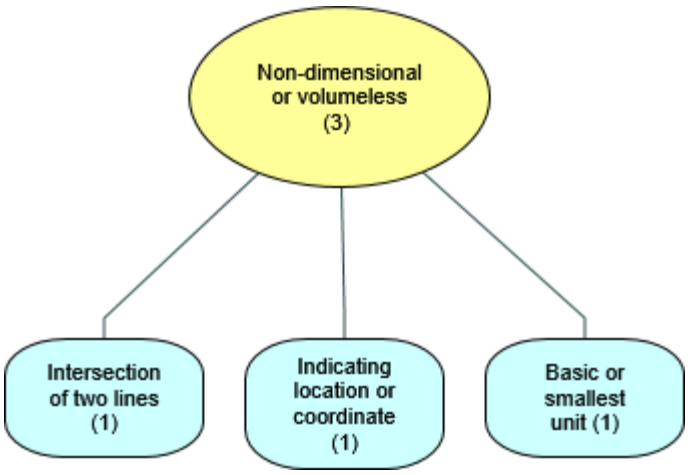

FIGURE 2b. *Description of a point with two keywords*

When Figure 2b is examined, the keywords used by 3 prospective teachers using two keywords to describe the point concept can be seen. In the description made by these 3 prospective teachers, the keyword 'non-dimensional or volumeless' was used jointly and this keyword was accompanied by the keywords 'intersection of two lines', 'indicating location or coordinate' and 'basic or smallest unit'. The description using the keyword 'Intersection of two lines' is "*It is non-dimensional. It is the intersection set of two lines that are not coincident.*" (OA07) while the description using the keyword 'Indicating location or coordinate' is "*The mark that does not cover a place in space but only indicates coordinate*" (OA29), whereas the description using the keyword 'Basic or smallest unit' is "*The smallest non-dimensional structure that forms geometric shapes"* (OA30).

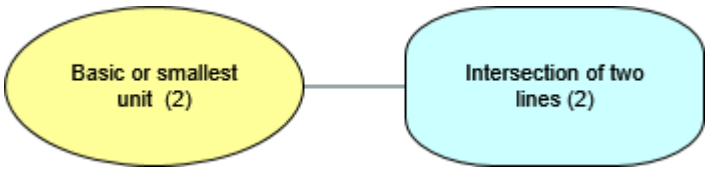

FIGURE 2c. *Description of a point with two keywords*

In Figure 2c, it is seen that 2 prospective teachers used the keyword 'Basic or smallest unit' together with the keyword 'Intersection of two lines' in order to describe the point concept. The descriptions in which such keywords are used is expressed as "*The smallest unit that is only one and which two lines intersect in space*" (OA37) and "*The smallest unit that allows all object in geometry to occur, it is the location where two lines intersect*" (OA61).

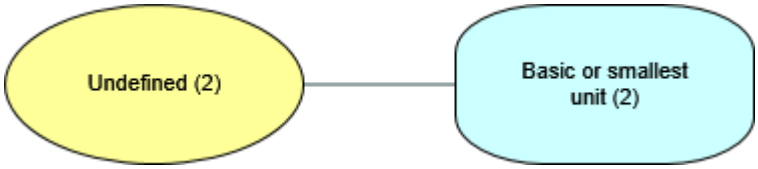

FIGURE 2d. *Description of a point with two keywords*

In Figure 2d, the keyword 'Basic or smallest unit' being the keyword that 2 prospective teachers used together with the keyword 'Undefined' to describe the point concept can be seen. The descriptions in which these keywords are used were expressed by the prospective teachers as "*The smallest sign being undefined*" (OA42) and "*The undefined shape that constitute the foundation of everything.*" (OA56).

Description of a point with three keywords

When Figure 3 is examined, the keywords used by 3 prospective teachers using three keywords to describe the point concept can be seen. The keywords 'Trace' and 'Undefined' were given place in all three descriptions while 3 prospective teachers were using different third keywords. As seen in Figure 3, these keywords are 'Non-dimensional or volumeless', 'Intersection of two lines' and 'Indicating location or coordinate'.

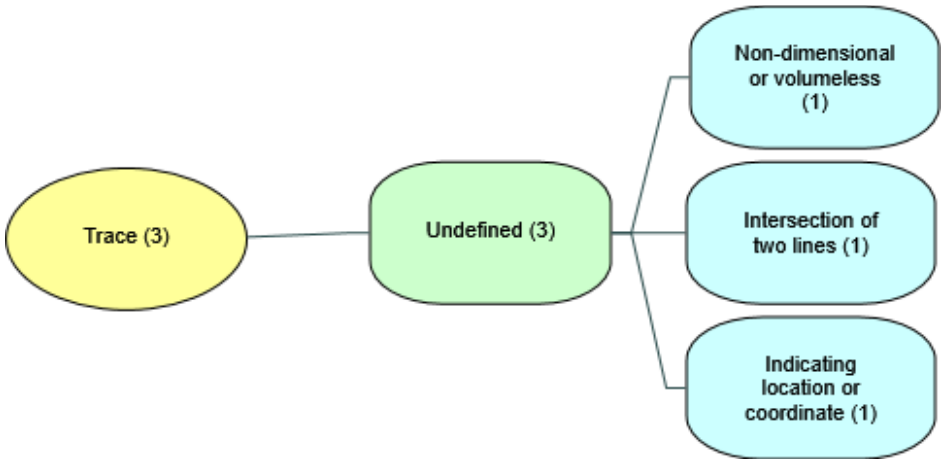

FIGURE 3. *Description of a point with three keywords*

The description made by the prospective teacher using the keyword 'Non-dimensional or volumeless' is "*Actually, point is one of the terms without definition*. *It is defined as the trace that the pen leaves on paper. The point is non-dimensional. In other words, it doesn't have width, length and thickness*." (OA04) while the description made by the prospective teacher who used the keyword 'Intersection of two lines' is "*I have not seen a full definition of the point but we can define it like this. It is formed by the intersection of at least two lines or it is the shape that is formed when we touch the tip of a pen*" (OA14). The point description made by the third prospective teacher is "*It doesn't have an exact definition. The trace left on the board with the tip of the pen, a geometric location where I can express the coordinates*" (OA36).

Description of a point with four keywords

In Figure 4, the figure formed by the four keywords used by two prospective teachers in the point description they make is presented. In these two descriptions, it is observed that the keywords 'Trace', 'Non-dimensional or volumeless' and 'Indicating location or coordinate' were included in both explanations while the fourth keyword used by prospective teachers was different. One prospective teacher used the keyword 'Basic or smallest unit' while the other prospective teacher used the keyword 'Undefined'.

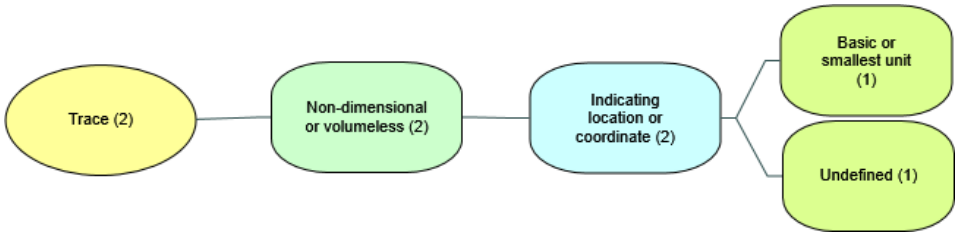

FIGURE 4. *Description of a point with four keywords*

The point description made by the prospective teacher using the keyword 'Basic or smallest unit' is "*The smallest geometric term with any coordinate but with a dimension in the plane. The shape that is formed when we press the pen lightly on the paper*" (OA11), while the description by the prospective teacher using the keyword "Undefined" is "*One of the undefined concepts in geometry. It is the trace left by the chalk on the board. A coordinate without volume in Euclidean space*" (OA 31).

Description of a Line Concept

The answers given by the prospective teachers to the question "Can you describe the line?" are examined and the keywords used for describing the line term within the answers are detected as 'Set of points', 'Unlimited or infinite', 'Direction or linearity', 'Two points', 'Straight line', 'Starting and ending points', 'Single dimension', 'Axis' and 'Slope'. While each participant is indicated on the horizontal axis of the graph given in Figure 5, the keywords used by each participant to describe the concept of point are indicated on the vertical axis.

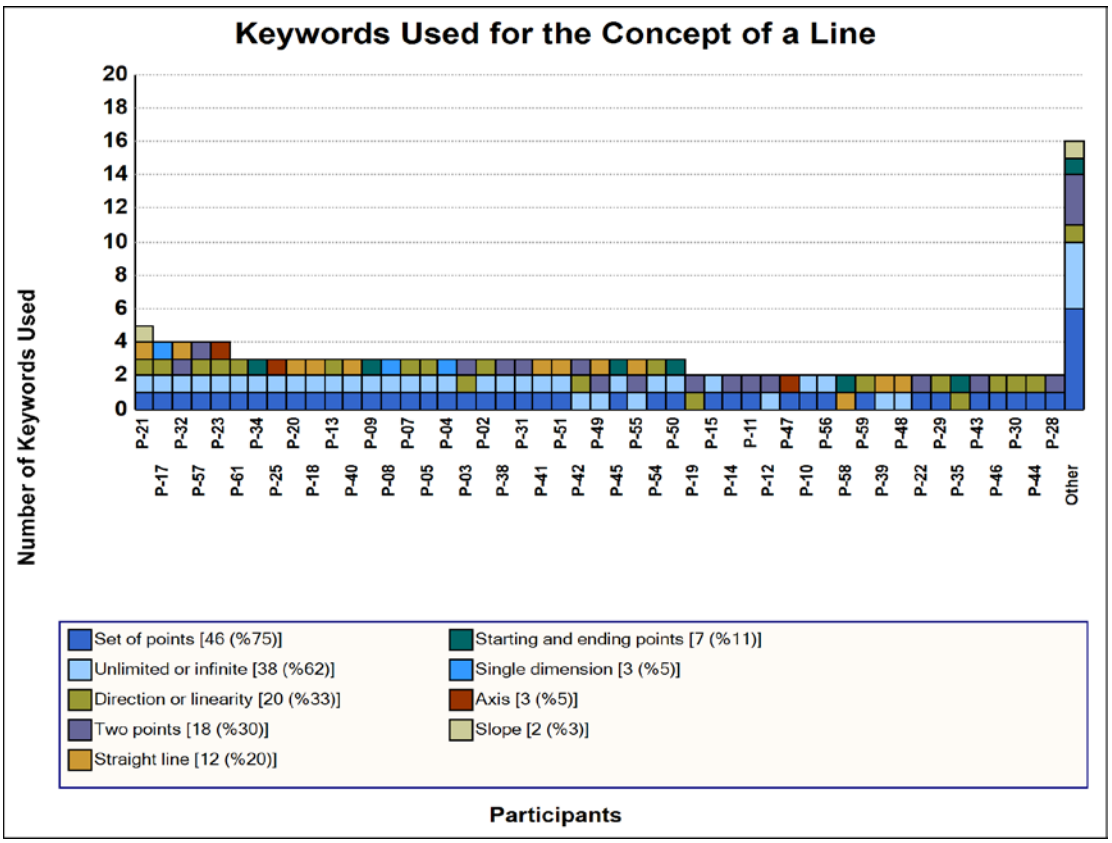

FIGURE 5. *Keywords used by prospective teachers to describe the concept of a line*

Given the keywords used by prospective teachers to describe the line concept; the keywords 'Straight line', 'Single dimension', 'Unlimited or infinite' and 'Starting and ending

points' are present in the essence of this concept. Moreover, since the keywords 'Axis', 'Direction or linearity', 'Set of points', 'Two points' and 'Slope' are the descriptions that emphasize both the physical appearance and features of the line concept, they belong to the correlation of this concept. On the other hand, while only the 'Straight line' is the pivot sign among these keywords that point at the concept of line, the other keywords used by prospective teachers are the mathematical sings that evoke the concept of line. These nine signs used by the prospective teachers when describing the line concept were used 149 times in total and, as seen in Figure 5, the number of signs used in each description are shown between one and four. The keywords accompanying the description of point are given under the following sub-headings.

Description of a line with one keyword

Three of 8 (13%) prospective teachers using one keyword used the keyword 'Two points'. The description made by one of these prospective teachers was "*One line passes through two points*" (OA16). Two of 8 prospective teachers used the keyword 'Set of points'. The description made by one of these prospective teachers is "*Line is a set of points*" (OA52). Two of 8 prospective teachers used the keyword 'Unlimited or infinite'. One of the descriptions containing this keyword is "*Line segment extending towards infinity*" (OA53). One of 8 prospective teachers made a description for the line concept by using the keyword 'Slope' and this explanation is "*The equation system in which the absolute value of the slope of each point we chose is constant*" (OA6).

Description of a line with two keywords

The analysis of the keywords in the descriptions made by 24 (39%) prospective teachers describing the line concept with two keywords are tried to be represented in Figure 6a, Figure 6b, Figure 6c and Figure 6d below.

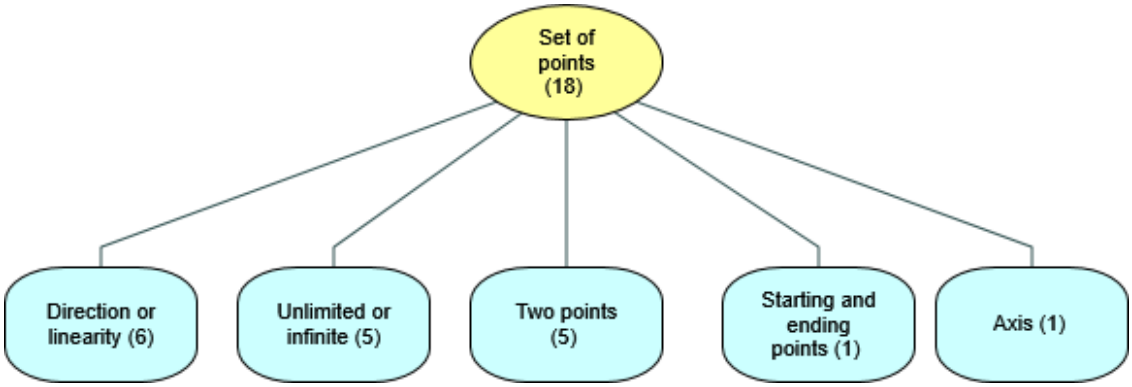

FIGURE 6a. *Description of a line with two keywords*

When Figure 6a is analyzed, it is seen that six of 18 prospective teachers used the keyword 'Set of points' together with the keyword 'Direction or linearity'. The descriptions made by two prospective teachers containing these two keywords are; "*We can define it as a set of linear points*" (OA27) and "*The set of linear points*" (OA29). It was observed that five prospective teachers used the keyword 'Set of points' together with the keyword "Direction or linearity". One of these descriptions is "*The shape formed by infinite number of dots is called line. The line goes to infinity*" (OA10). It was seen in the descriptions by prospective teachers that the concept of infinity is used in the sense of infinite number of points and a set of points extending towards infinity, namely in the sense of indicating location. Another keyword used with the keyword 'Set of points' was the keyword 'Two points'. Two descriptions made by prospective teachers using these keywords are "*A set of points connecting two points*" (OA11) and "*A set of points of which two points are evident*" (OA22). The keywords 'Axis' and 'Starting and ending points' were used once by the prospective teachers in the same description containing the keyword 'Set of points'. These descriptions were "*It is a set of points that meet on the same principal axis*" (OA10) and "*The set of points with an uncertain start and end point*" (OA01).

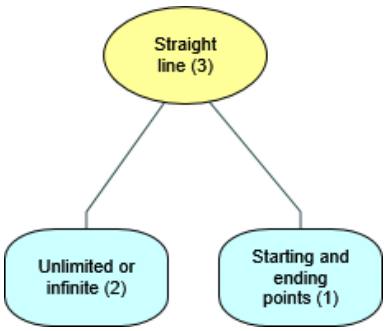

FIGURE 6b. *Description of a line with two keywords*

In Figure 6b, it is seen that two of 3 prospective teachers used the keyword 'Straight line' together with the keyword 'Unlimited or infinite'. Descriptions by these two prospective teachers are "*It is a straight line that extends to infinity at both ends and contains the numbers as well*" (OA 39) and "*It is a straight line with two ends extending to infinity*" (OA 48). One of the 3 prospective teachers used the keyword 'Straight line' together with the keyword 'Starting and ending points' in the same description. This description was "*A smooth straight line with no certain starting and ending points*" (OA58).

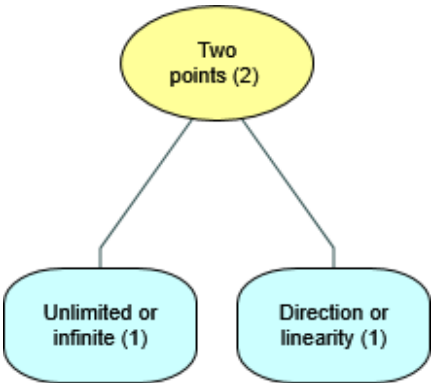

FIGURE 6c. *Description of a line with two keywords*

In Figure 6c, it is seen that one of 2 prospective teachers used the keyword 'Two points' together with the keyword 'Unlimited or infinite'. The description made by this prospective teacher is "*It is a geometric concept that connects any two points on which we can choose infinite points and extends to infinity at both ends*" (OA12). In the description made by the other prospective teacher, the keywords 'two points' and 'Direction or Linearity' are given place in the same description. This description was "*The line gets formed when we connect at least two points that are linear*" (OA19).

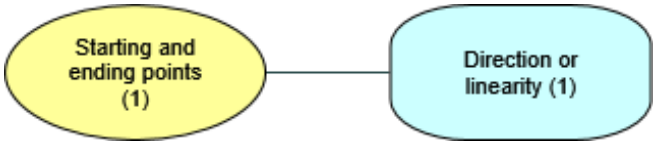

FIGURE 6d. *Description of a line with two keywords*

Figure 6d shows the keywords used by a prospective teacher to describe the concept of line. These keywords are 'Starting and ending points' and 'Direction or linearity' and the description by the prospective teacher is *"Is that having a starting point, direction and orientation"* (OA35).

Description of a line with three keywords

The analysis of the keywords in the descriptions made by 24 (39%) prospective teachers describing the line concept with three keywords are tried to be represented in Figure 7a and Figure 7b below.

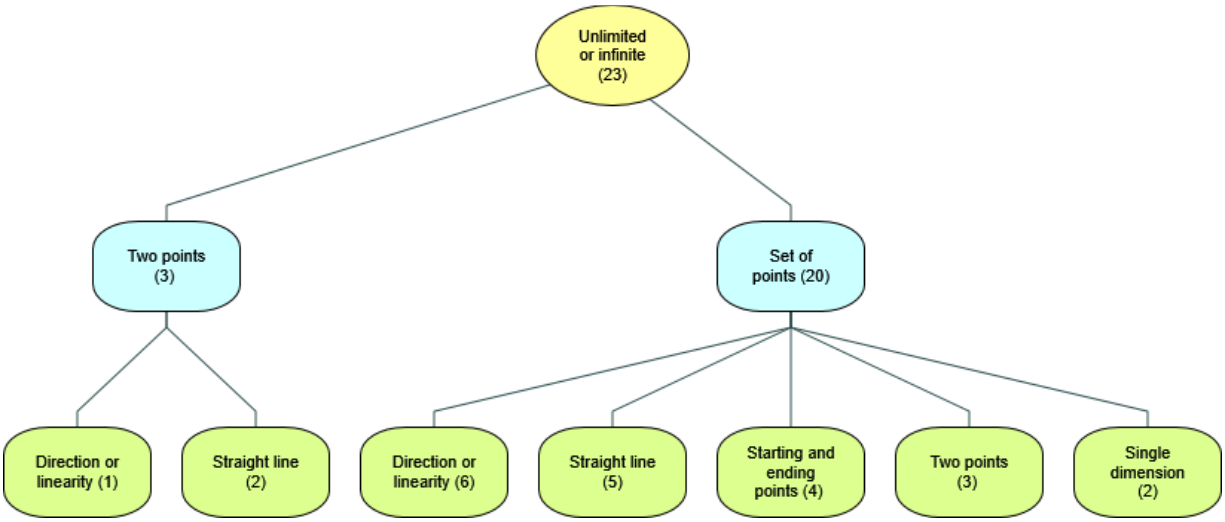

FIGURE 7a. *Description of a line with three keywords*

When Figure 7a is examined, it is seen that 20 of 23 prospective teachers who use three keywords to describe the concept of line select the keyword 'Set of points' as the second keyword, while the other 3 prospective teachers chose the keyword 'Two point' as the second keyword. In their descriptions, two of the 3 prospective teachers using the words 'Unlimited or infinite' and 'two points' used the keyword 'Straight line'. One of these descriptions is "*Straight line without borders passing through two points*" (OA55). On the other hand, a prospective teacher included the keyword 'Direction or linearity' in his description. This description is presented as "*Extending the direction of two points to infinity by connecting them*" (OA 42). 6 of 20 prospective teachers who used the keywords 'Unlimited or infinite' and 'Set of points' in their descriptions included the keyword 'Direction or linearity' as the third keyword. Two of these explanations are "*The fact that is formed by joining points linearly and extending infinitely.*" (OA61) and "*The set of points which infinite number of points form by coming together in the same direction*" (OA05). 5 of 20 prospective teachers used the keyword 'Straight Line' as the third keyword. Two of these descriptions are "*A set of points on a unlimited straight line"* (OA40) and "*It is formed by joining the points on a straight line with two unlimited ends*" (OA41). 3 of 20 prospective teachers used the keyword 'Two points' as the third keyword. A description made by one of the prospective teachers is "*A set of points of infinite length passing through at least two points*" (OA31). 4 of 20 prospective teachers used the keyword 'Starting and ending points' as the third keyword. The description made by one of the prospective teachers was "*The combination of a set of points without a starting and ending point going to infinity is called as line*" (OA09). Finally, 2 of 20 prospective teachers used the keyword 'Single dimension' as the third keyword. A description made by one of the prospective teachers is "*It is the shape that is formed by infinite number of points that come side by side. It is one-dimensional. It has no width and thickness. It only has a length that extends to infinity.*"(OA04).

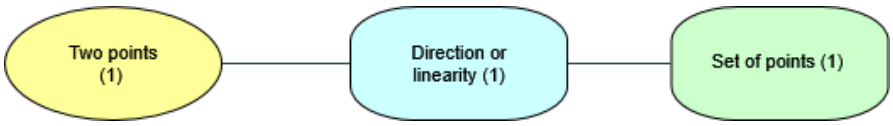

FIGURE 7b. *Description of a line with three keywords*

Figure 7b shows three keywords used by a prospective teacher to describe the concept of line. These keywords are 'Two points', 'Direction or linearity' and 'Set of points', whereas the description made by the prospective teacher is "*It is the set of linear points on the plain. The must be at least two points to form a line*" (OA03).

Description of a line with four keywords

When Figure 8 is examined, the keywords used by 4 prospective teachers using four keywords to describe the line concept can be seen. It was observed that the keywords 'Set of points' and 'Unlimited or infinite' were included in the descriptions by 4 prospective teachers, whereas these prospective teachers made different choices while using two other keywords.

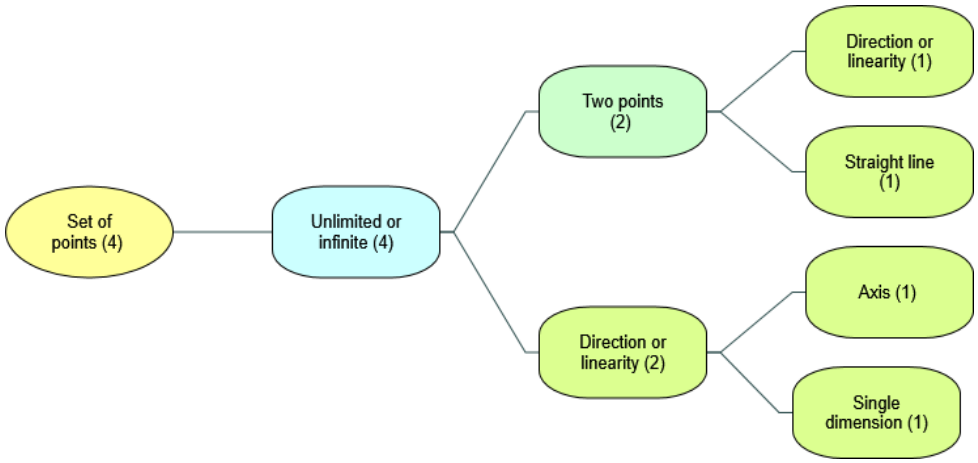

FIGURE 8. *Description of a line with four keywords*

In addition to the 'Set of points' and 'Unlimited or infinite' keywords, one of the two prospective teachers who used the 'Two points' keyword in his description used the keyword 'Direction or linearity'. The description by this prospective teacher is "*It is a set of linear points that connect two points and go to infinity*" (OA57), and the description made by the prospective teacher using the fourth keyword as 'Straight line' is "*The straight line formed by joining of infinite number of points. Only one line passes through any two points taken*"(OA05). In the descriptions about the line concept by the other two prospective teachers, the keyword 'Direction or Linearity' is used in common as the third keyword, while the fourth keywords are again different. Among these, the description using the keyword 'Axis' is expressed as "*The set of points formed by the infinite number of points coming next to each other along the same axis (linear)*" (OA23), while the description using the keyword 'Single dimension' is expressed as "*It is the single dimension expression formed by infinite number of points in the same direction* (OA17).

Description of a line with five keywords

Figure 9 shows five keywords used by a prospective teacher to describe the concept of line. These keywords are, respectively, 'Set of points', 'Unlimited or infinite', 'Direction or linearity', 'Straight line' and 'Slope', and the description by the prospective teacher is "*Each x, y real number (x,y) element indicates point for IRxIR. It is the linear straight line formed by infinite number of points. It has a slope*" (OA21).

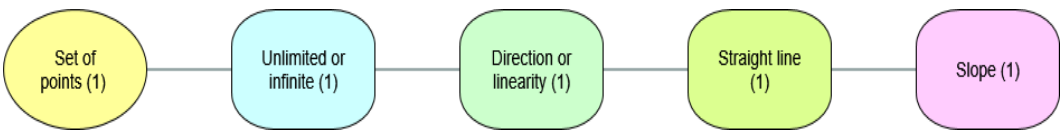

FIGURE 9. *Description of a line with five keywords*

Description of a Plane Concept

The answers given by the prospective teachers to the question "Can you describe the plane?" are examined and the keywords used in the description of plane term within the answers are detected as 'Region, area, shape or surface', 'Two lines', 'Dimension', 'Set of points', 'Set of lines', 'Coordinate or axis', 'Three points', 'One point and one line', 'Non-parallel or three intersecting lines' and 'Undefined'. While each participant is indicated on the horizontal axis of the graph given in Figure 10, the keywords used by each participant to describe the concept of plane are indicated on the vertical axis.

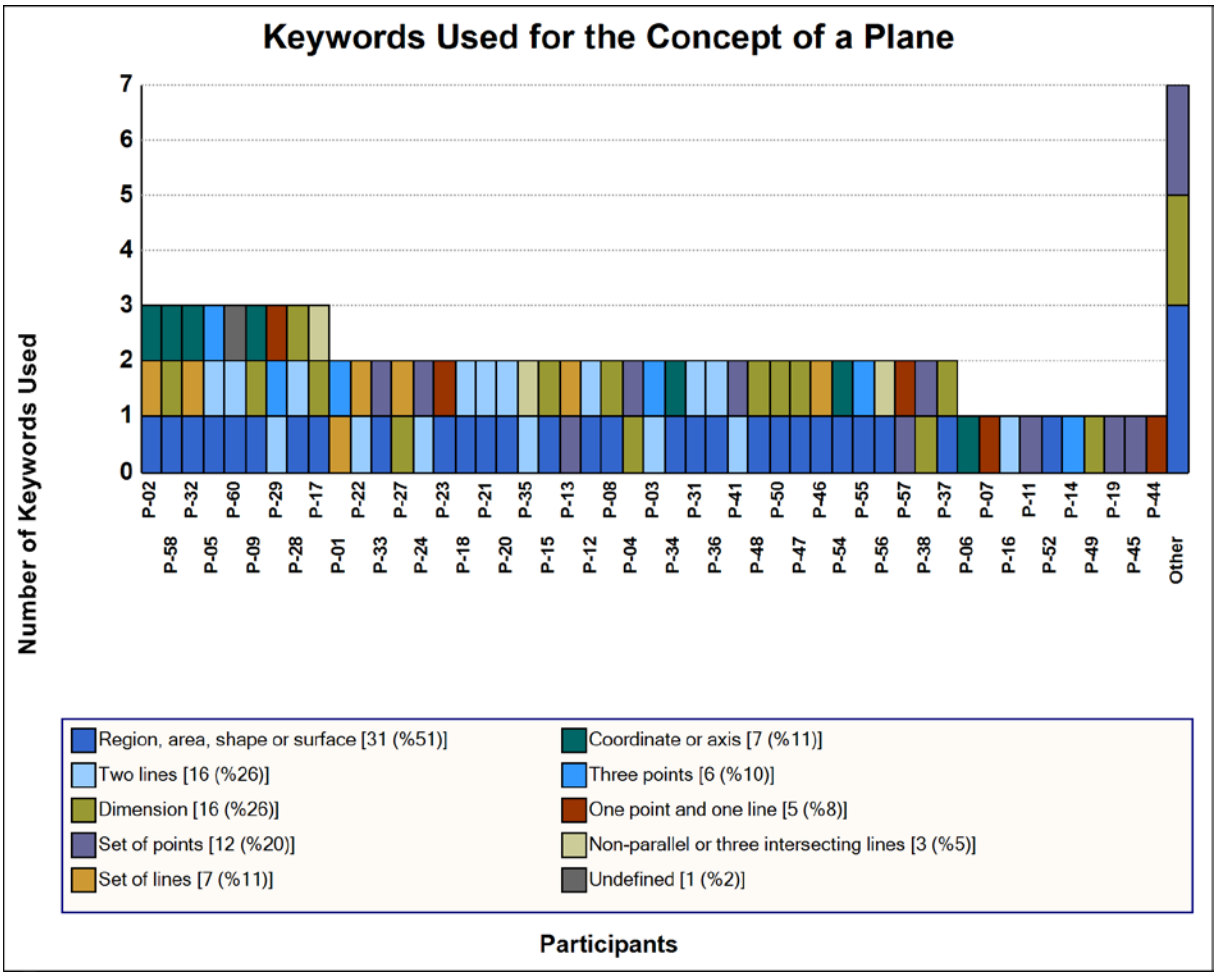

FIGURE 10. *Keywords used by prospective teachers to describe the concept of a plane*

When the descriptions made by prospective teachers for the concept of plane are taken into consideration, it is seen that they use the keywords regarding both the essence and the concept as they have done while describing point and line. While 'Dimension', 'Undefined' and 'Region, area, shape or surface' are the descriptions with regard to the essence of the plane concept, 'Coordinate or axis', 'Two lines', 'Set of points', 'Three points', 'Set of lines', 'One point and one line' and 'Non-parallel or three intersecting lines' keywords are the keywords with regard to the physical appearance or properties of plane. Among these ten keywords given place in the descriptions regarding the plane concept, only the 'Region, area, shape or surface' sign is determined to be a pivot sign, while the other nine keywords are mathematical signs that refer to plane concept. These ten signs used by the prospective teachers were used 104 times in total and, as seen in Figure 10, the number of signs used in each description are shown between one and three. The keywords accompanying the description of point are given under the following sub-headings.

Description of a plane with one keyword

5 of 17 (28%) prospective teachers using one keyword used the keyword 'Set of points' in the descriptions they made. The description made by one of these prospective teachers was "*The plane is the set of infinite number of points*" (OA11). The keyword 'Region, area, shape or surface' was used as a single keyword by 4 prospective teachers. One of these descriptions is "*Each separated area in space is called a plane*" (OA25). 3 of 17 prospective teachers used the word 'Dimension' in their descriptions, and one of these descriptions was "*The two-dimensional fact with a width and length but no height*" (OA61). The keyword 'One point and one line' was used as a single keyword by 2 prospective teachers in their descriptions. An example description is expressed as "*It is the shape formed by a straight line and a point that is not on that line*" (OA07). On the other hand, the keywords 'Three points', 'Coordinate or axis' and 'Two lines' are used once as a single keyword to describe the concept of plane. The example description using the keyword 'Three points' is "*The set that is formed by at least three points or line*" (OA14), whereas the example description using the keyword 'Coordinate or axis' is "*Each number range between x and y axes in the Cartesian coordinate system in which a transaction is made is called as plane"* (OA06). An example description using the keyword 'Two lines' is expressed as "*Two lines intersecting at one point indicates a plane*" (OA16).

Description of a plane with two keywords

The analysis of the keywords in the descriptions made by 30 (49%) prospective teachers describing the plane concept with two keywords are tried to be represented in Figure 11a, Figure 11b, Figure 11c and Figure 11d below.

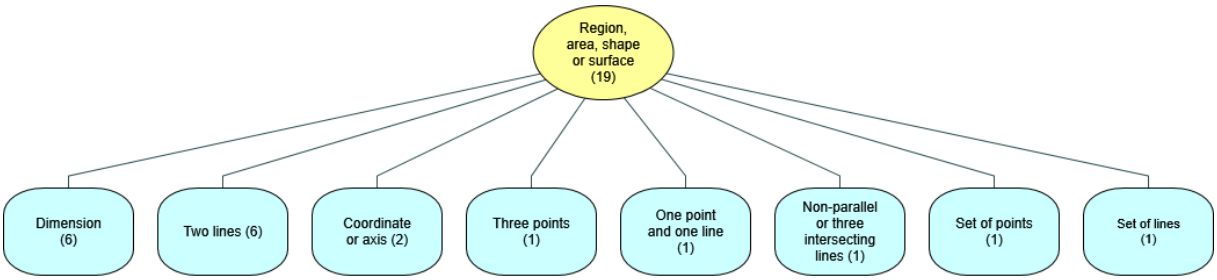

FIGURE 11a. *Description of a plane with two keywords*

When Figure 11a is analyzed, it is seen that six of 19 prospective teachers used the keyword 'Region, area, shape or surface' together with the keyword 'Dimension'. The descriptions by two prospective teachers using these two keywords are given as "*The visible upper surfaces of three-dimensional objects such as table, notebooks are called like that. It is the two-dimensional member of space*" (OA48) and *"All shapes in space having length and width but no depth"* (OA37). 6 prospective teachers used the keyword 'Region, area, shape or surface' together with the keyword 'Two lines' and two of these descriptions are *"The area between two lines and the area scanned as a result of their intersection is called plane"* (OA18) and "*It is the area formed by the intersection of at least two lines*" (OA20). 2 prospective teachers included the keyword 'Coordinate or axis' together with the keyword 'Region, area, shape or surface' in their descriptions. One of these descriptions is "*The region formed by coordinates such as (x, y), (x, z), (y, z)*" (OA54). The keywords 'Three points' and 'One point and one line' are used together with the keyword 'Region, area, shape or surface' once in the same description, and these descriptions are, respectively, "*The shape that three non-linear points have formed*" (OA55) and "*The place where one point and a line can be found is called a plane*"(OA23). Similarly, the keywords 'Non-parallel or three intersecting lines', 'Set of points', and 'Set of lines' were used once together with the keyword 'Region, area, shape or surface'. Example descriptions are, respectively, as follows; "*The area formed by at least three lines intersecting with each other*" (OA56), "*It is the area consisting of infinite number of points*" (OA33) and "The area formed by the

joining of lines, whereas such lines are formed in space by many points coming together adjacently without any space between them" (OA46).

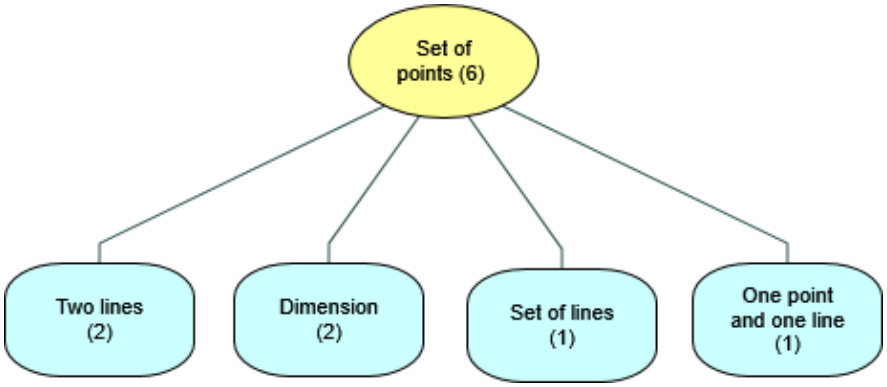

FIGURE 11b. *Description of a plane with two keywords*

Figure 11b shows the distribution of the second keywords used together with the keyword 'Set of points' by 6 prospective teachers to describe the plane concept. 2 prospective teachers used the keywords 'Two lines' in their descriptions, and these descriptions are "*The set formed by the combination of infinite number of points. One plane passes through two intersecting lines*" (OA24) and "*It is the set of points between the lines formed by two parallel lines*" (OA41). The use of the 'Two lines' keywords is different in the two descriptions. In the first description, this keyword means that two lines indicate one plane, while in the second description it is used as a line that limits the points of plane. Likewise, 2 prospective teachers used the keyword 'Dimension' this time together with the keyword 'Set of points', and the example description is "*Plane also gets formed by the joining of infinite number of points just like the line. Plane is also two-dimensional. It has only width and length. It has no thickness*" (OA04). In addition, the keywords 'Set of lines' and 'One point and one line' were used once together with the keyword 'Set of points' by the prospective teachers. The plane descriptions using these keywords are expressed as "*A term containing infinite number of points, lines*" (OA13), for the 'Set of lines' and as "*A set of infinite number of points that contain a point and a line at the same time*" (OA57) for 'One point and one line'.

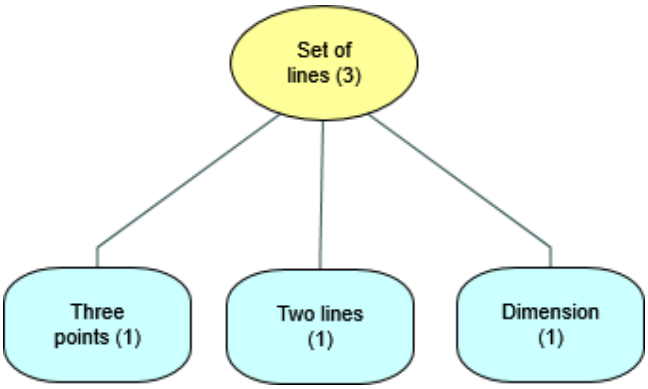

FIGURE 11c. *Description of a plane with two keywords*

When Figure 11c is examined, it is seen that 3 prospective teachers used the keywords 'Three points', 'Two lines' and 'Dimension' once together with the keyword 'Set of lines' to explain the plane concept. An example description using the keyword 'Three points' together with the keyword 'Set of lines' is *"It specifies three nonlinear points, or the set of lines parallel to each other and in the same direction forms planes"* (OA01). The example description using the words 'Two lines' is expressed as *"All lines passing through two overlapping lines in Euclidean*

space" (OA22), while the description using the keyword 'Dimension' is expressed as "*It is the set of lines extending to infinity transversely and longitudinally*" (OA27).

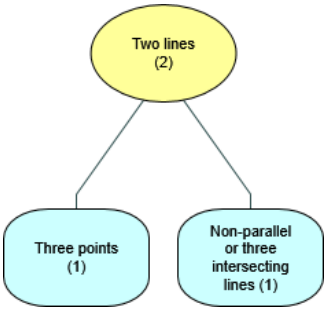

FIGURE 11d. *Description of a plane with two keywords*

To describe the concept of plane in Figure 11d, it was seen that 2 prospective teachers preferred to use 'Three points' and 'Non-parallel or three intersecting lines' keywords in their second keywords together with the keyword 'Two lines'. The description in which the keywords 'Two lines' and 'three points' are used together is "*A set of points containing at least three points formed by two lines intersecting at one point. In order to specify a plane, there must be at least three points*" (OA03), while the description in which the keywords 'Two lines' and 'Non-parallel or three intersecting lines' are used together is "*The set formed by the intersection of at least three lines. Two parallel lines specifies a plane*" (OA35).

Description of a plane with three keywords

The analysis of the keywords in the descriptions made by 9 (2%) prospective teachers describing the plane concept with three keywords are tried to be represented in Figure 12a and Figure 12b below.

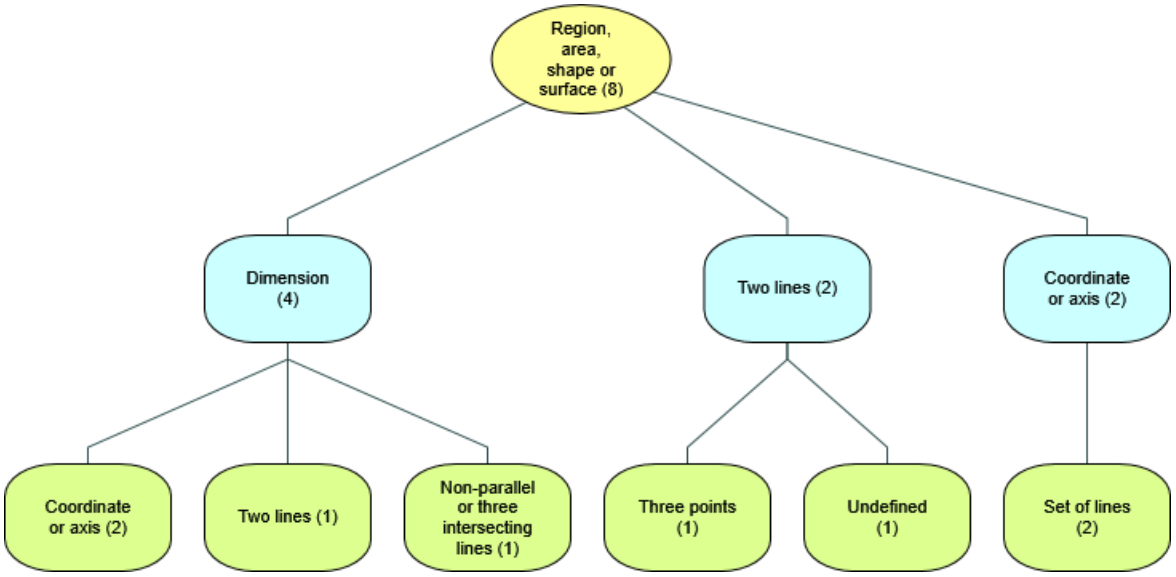

FIGURE 12a. *Description of a plane with three keywords*

When Figure 12a is examined, it is seen that 4 of 8 prospective teachers who use three keywords to describe the concept of plane select the keyword 'Dimension' as the second keyword, while the other 2 prospective teachers chose the keyword 'Coordinate or axis' and the other 2 prospective teachers chose the keyword 'Two lines' as the second keyword. When the third keywords used by the prospective teachers in their descriptions were examined, it is seen that 2 of the prospective teachers who used the keywords 'Region, area, shape or surface' and

'Dimension', used the keyword "Coordinate" or "axes" as the third keyword for the plane concept. One of these descriptions is "*The area formed by intersection point of the origin and coordinates on the x and y coordinate plane is called a plane. It is 2-dimensional*" (OA09). A prospective teacher included the 'Two lines' keyword as the third keyword to describe the concept of plane. The explanation made by this prospective teacher is *"The two-dimensional shape formed by at least two lines"* (OA28). Similarly, a prospective teacher used the keyword 'Non-parallel or three intersecting lines' as the third keyword. The description made by this prospective teacher is "*The surface of the board, the table top, etc. is the surface formed by three intersecting lines. It is a 2-dimensional structure formed by at least three non-parallel lines*" (OA17). One of the two prospective teachers using the keywords 'Region, area, shape or surface' and 'Two lines' in the description of plane used the keyword 'Three points' as the third keyword, while the other one used the keyword 'Undefined'. The plane description, which includes the keyword 'Three points' is *"The region that is not in the same direction and formed by three points. The region formed by two non-overlapping lines"* (OA05), while the description containing the keyword 'Undefined' is "*It is an undefined term*. *It is the plane between two parallel lines"* (OA60). The 2 prospective teachers who used the keywords 'Region, area, shape or surface' and 'Coordinate or axis' for the plane description used the keyword 'Set of lines' as the third keyword. The example description was made by the teacher candidate as "*The region formed by the lines passing through two or three of the x, y, z axes*" (OA02).

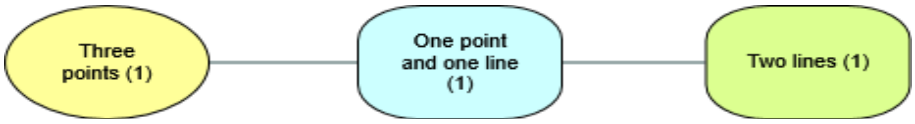

FIGURE 12b. *Description of a plane with three keywords*

Figure 12b shows three keywords used by a prospective teacher to describe the concept of plane. These keywords are, respectively, 'Three points', 'One point and one line' and 'Two lines' and the description made by the prospective teacher is *"The set formed by two intersecting lines, one line and one point or by three non-linear points"* (OA29).

DISCUSSION and CONCLUSION

As a result of the findings obtained from the research, it was determined that prospective teachers used six keywords to describe the concept of point, nine keywords to describe the concept of line, and ten keywords to describe the concept of plane. The concepts that prospective teachers try to describe are primitive concepts and the descriptions of these concepts given by Euclid being hard to understand on secondary education level have forced the teachers to use different keywords to describe these concepts. Therefore, it was observed that prospective teachers used the keywords co-related to primitive terms in addition to the keywords related to the essence while making descriptions of primitive terms. Although such an approach is not mathematically suitable for defining a concept, they state that, in accordance with the approach of classical logicians, the descriptions related to the correlation will be definitions along with the definitions related to the essence (Öner, 1986). On the other hand, when the descriptions by prospective teachers about primitive terms are examined from the framework of semiotics mediation theory, it is observed that the descriptions made by prospective teachers that describe the geometric shapes substituting these terms in the descriptions about these primitive terms, the models referring to these terms in daily life, or the pivot and mathematical signs such as the symbols that point to these terms were employed simultaneously. Therefore, as in this study, revealing the signs that occur in the minds of individuals is important in order for the classroom activities to be designed from the perspective of semiotic mediation theory and to become effective (Turgut, 2015, 2017). However, since these sings cannot always represent the concept they are trying to describe, it should be noted

that they cannot replace but refer to that concept (Duval, 2006; Sáenz-Ludlow, & Presmeg, 2006; Short, 2007).

When the descriptions in Euclid's Elements with regard to the point are taken into consideration, it is seen that the keywords 'Undefined' and 'Non-dimensional or volumeless' are in the essence of this concept and the prospective teachers in this study use these two keywords as signs when describing the point. The descriptions using such essential signs are found in various studies (Dane, 2008; Dane & Başkurt, 2012; Tuluk, 2014). For example, in the study by Tuluk (2014), the expression "Point is zero-dimensional" is stated as a description for the point concept.

On the other hand, when the prospective teachers failed to reach the definition related to the essence of undefined concepts, it was seen that they made descriptions related to the concept either by focusing on its physical appearance or on the features related to this concept. In this context, while 'Trace' and 'Indicating location or coordinate' keywords are the signs used by prospective teachers based on the physical appearance of the point, the signs such as 'Basic or smallest unit' and 'Intersection of two lines' are focused on the properties of the point and used by prospective teachers to describe this concept. In case where making descriptions regarding the essence is not possible, since it is believed that prospective teachers feel the need for a qualitative description of these concepts and therefore they prefer using the signs regarding the correlation in their explanations.

It is seen that, when Euclid's descriptions about the line in his famous book Elements are taken into consideration, the keywords 'Straight line' and 'Single dimension', on the other hand, as a result of the description made by Altun (2005a) suggesting that the line extends to infinity at both ends, the keywords 'Unlimited or infinite' and 'Starting and ending points' are in the essence of this concept and that the prospective teachers in this study made descriptions about the essence for the line concept by using these signs when describing the line. On the other hand, it is determined that the prospective teachers made descriptions by emphasizing the physical appearance or characteristics of the line when they did not prefer the keywords accompanying the descriptions about the essence. In this context, the 'Axis' sign is the keyword used by prospective teachers based on the physical appearance of the line, while the 'Direction or linearity', 'Set of points', 'Two points' and 'Slope' signs are the keywords used by focusing on the features of the line. The descriptions using such signs regarding the correlation or the physical appearance of line are found in various studies (Kiriş, 2008; Öksüz, 2010). For example, in a study by Öksüz (2010), the expression "set of unlimited set of straight points is line" is suggested as a description made for the line concept. Similarly, in the study where Dane and Başkurt (2012) classified student views according to their perception levels, the descriptions of the line concept are expressed as "a straight line with an infinite length, a straight line with both ends extending to infinity", "a set of linear points" and "a straight line".

Likewise, considering the descriptions in Euclid's Elements about the plane, it is seen that the keywords 'Dimension', 'Undefined' and 'Region, area, shape or surface' are in the essence of this concept and prospective teachers in this study described the plane by using these signs. On the other hand, it is seen that the prospective teachers have taken into consideration the 'Coordinate or axis' sign, physical appearance while making correlation description regarding the plane and have preferred the signs 'Two lines', 'Set of points', 'Three points', 'Set of lines', 'One point and one line' and 'Non-parallel or three intersecting lines' by focusing on the properties of the plane. In the study by Dane and Başkurt (2012), it is seen that similar signs are used to describe the plane. For example, expressions such as 'a flat area, region', 'the wall surface and the floor or ceiling of the class' and 'the thing with infinite width and length on which a line, point and ray may be available' and 'coordinate plane' are indicated as student descriptions regarding the plane.

In addition, these findings obtained, contrary to the results of Ubuz and Gökbulut's (2015) study on the descriptions made by prospective class teachers regarding the pyramid concept, demonstrates that the prospective teachers in this study used mathematical language well in their descriptions to define point, line and plane, whereas they included both pivot and

mathematical signs pointing at these concepts and made use of the features of these concepts in order to create definition.

As a result, in this study, it is observed that at least one, at most four different keywords are used in the descriptions of point concept, and at least one, at most five keywords are used in the descriptions of line concept, and at least one and at most three keywords are used in the descriptions of plane concept. Therefore, it is determined that the prospective teachers used more than one sign and included them in their descriptions by focusing on the properties, appearances and symbols of undefined concepts. However, according to Van Dormolen and Zaslavsky (2003) in order for the concept defined to exist; necessary and sufficient conditions must be specified and excessive conditions must not be used. In this study, however, when some descriptions made by prospective teachers are taken into consideration, it is found that both essential and correlational signs were used together in the same description. Although this situation may seem controversial in terms of being economical when viewed by Van Dormolen and Zaslavsky (2003); Çakıroğlu (2013) did not see any harm in emphasizing more than necessary features in order for a description to be understandable in learning environments. Furthermore, Poincaré (1909/2009) suggests that what is important for a good definition in terms of teaching is that it is easy to understand by students rather than the principle of economics. Therefore, when the importance of Euclid's primitive (undefined) terms are taken into consideration, it is important that prospective mathematics teachers can think flexibly enough regarding this matter. For this reason, as Leikin and Winicki–Landman (2000) suggested, it is important that prospective teachers know the properties that the definition should have and have full control over the different descriptions of the respective concet when necessary in order to make the definition of a concept understandable. On the other hand, when the descriptions made by prospective teachers about point, line and plane in this study are taken into consideration, even though it is seen that the signs used for these concepts (model, shape, feature, symbol, etc.) override the concept itself in some cases, it should be noted that these signs used cannot always represent the concept that they intend to explain and may not be used instead of the real concept.

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