-- BEST PRACTICES --

Developing teaching materials supported with Geogebra for circle and disc subject at seventh grade[1]

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Abstract: Many studies showed that computer-assisted teaching had positive effects on students' success, attitude, permanent and motivation. On the other hand, some studies showed that students had various difficulties and some misconceptions about circle and disc subject. It is recommended to use computer technology effectively in the modeling of geometric concepts and the discovery of relationships. In this context, computer-assisted teaching material supported with GeoGebra software for "Circle and Disc" subject has been developed. The aim of this study is to introduce teaching material supported with GeoGebra for "Circle and Disc" subjects at seventh grade. The teaching material was carried out with 22 seventh grade students in a secondary school in Isparta. After the classroom application, teaching material supported with GeoGebra was revised in accordance with the opinions of teachers and students. As a result of the application, it was determined that teaching material supported with GeoGebra provides the opportunity to observe and discover in a dynamic environment, visualize the concepts and offers the opportunity for conceptual learning.

INTRODUCTION

Teaching methods and instructional materials play an important role in mathematics teaching. Baki (2008) stated that traditional teaching methods and tools which are preferred in geometry teaching which includes abstract thinking and reasoning are insufficient in conceptual learning and permanence. In the literature, it is emphasized that visualization and dynamics emphasized in teaching geometry overcome these difficulties (Berger, 2011; Saha, Ayup & Tarmizi, 2010). As a matter of fact, the good design and application of instructional materials that support different sensory organs and enable the student to actively explore information rather than memorizing information, help the students to become interested and motivated to the lesson and also increase the permanence of learning (Bahadır & Demir, 2017; Kutluca & Birgin, 2007; Yanpar Yelken, 2011). The use of materials in the teaching process facilitates the teaching of the subject for teachers, provides the opportunity for students to experiment and observe, question and explore, and provides collaborative learning opportunities with group activities (Birgin, Bozkurt, Gürel & Duru, 2015; Er & Sağlam Kaya, 2017; Kösa & Kalay). In this context, it is emphasized in the mathematics curriculum (MEB, 2013) that concrete and computer-assisted teaching materials should be used in the teaching process. Today, in addition to the use of many types of materials in terms of structuring information and increasing the permanence in learning, the use of computer-assisted teaching (CAI) materials has gained importance due to its features such as activating the student, providing a visual and dynamic learning environment through sound,

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image and animation, and providing many opportunities for experimentation and observation. Thus, the lack of computer technology in learning environments in our age, where rapid technological developments occur, creates an inconsistency between students' real life and learning environments (Powers & Blubaugh, 2005). However, it is important that the materials to be developed and preferred should be appropriate for the students' readiness, teacher qualifications and the structure of the subject to be taught. In this context, there is a need for CAI materials that provide students with the opportunity to observe, make assumptions, discover and construct information. In recent years, the use of GeoGebra, one of the dynamic geometry softwares (DGS) in mathematics/geometry teaching in our country, is increasing in use among researchers, teachers and students. The growth reasons of GeoGebra usage have been showed that every person is able to use easily even a beginner computer user at each level of mathematics instruction from elementary school to university, it includes in both the features of computer algebra system and CAI features together and has Turkish translation as well as free open source.

The use of GeoGebra, one of the dynamic geometric software in the teaching of geometry, gives students the opportunity to make many experiments and observations through visuals and animations, to embody geometric shapes in the mind, to discover geometric concepts, features and relationships (Baki, Kösa & Güven, 2011; Shadaan & Leong, 2013; Topuz & Birgin, 2020; Zengin & Tatar, 2015). On the other hand, various researches show that students have difficulty in learning "Circle and Disc" (Bekdemir, 2012; Güngörmüş, 2002; Kaygusuz 2011; Özsoy & Kemankaşlı, 2004; Yenilmez & Demirhan, 2013). In this respect, it is important to develop GeoGebra supported teaching materials for the teaching of discs and circles. For this reason, within the scope of this research, GeoGebra-supported teaching material containing GeoGebra activities and worksheets subject have been developed and applied for teaching seventh grade "Circle and Disc" in the classroom. In order to take our education system one step further it is thought that every research to be carried out with technology-enhanced instructional materials will encourage the use of technology by mathematics teachers. Therefore, the aim of this study is to introduce GeoGebra-supported teaching material developed for teaching the seventh grade "Circle and Disc" subject.

METHOD

In this research, design based research method has been used because of the development of GeoGebra supported teaching material. Design based research is a new research approach based on the revision of teaching strategies, materials and tools used in the process according to the findings obtained from the actual application process (Collins, Joseph, & Bielaczyc, 2004). For this reason, in this study, it has been discussed the development stages of the computer-assisted teaching material which has been used GeoGebra software on 7th grade "Circle and Disc", the characteristics of the activities and the process of pilot application in the classroom.

Design and Application Process of GeoGebra-Supported Teaching Material

In the process of developing GeoGebra-supported teaching material for teaching the seventh grade "Circle and Disc" subject, the stages are as follows:

- In the process of developing instructional materials, firstly, it has been examined whether there is a difficulty in learning "Circle and Disc". As a result of the literature review, it has been determined that students had difficulty in learning "Circle and Disc" and had some misconceptions (Bekdemir, 2012; Güngörmüş, 2002; Kaygusuz 2011; Özsoy & Kemankaşlı, 2004; Yenilmez & Demirhan, 2013; Yılmaz, Keşan & Nizamoğlu, 2013; Yılmaz, Keşan & Nizamoğlu, 2013; Yılmaz, Keşan & Nizamoğlu, 2001). In this direction, it is determined that there is a need for the development of instructional material for teaching this subject.
- The seventh grade mathematics curriculum has been examined and it has been searched to prepare a suitable instructional material by reviewed the related learning outcomes and objectives according to the concepts and characteristics of the subject.
- In the related literature, CAI research related with "Circle and Disc" subject (Özdemir & Tabuk, 2004) and the DGS of Cabri and GSP (Bağçıvan, 2005; Deniz & Özdemir Erdoğan,

2012; Kaplan & Öztürk, 2014), GeoGebra used as a computer supported teaching material (Balcı Şeker & Erdoğan, 2017; Bulut, 2013; Shadaan & Leong 2013; Uzun, 2014) were analyzed. As a result of literature review, it has been concluded that CAI material should be developed in teaching this subject.

- It has been decided that the use of GeoGebra among the DGSs would be more appropriate, since Turkish translation and free access were available during the development of CAI material, and in recent years it has been preferred among researchers, teachers and students. In the process of designing the CAI material activities, we will learn each of the videos how to teach each functions of "Circle and Disc" subject in 7th grade mathematics course from the GeoGebra official website (www.geogebra.org) and the videos with open use (www.youtube.com) about how to build the circle and disc dynamic materials. CAI GeoGebra activities have been designed to enable students to explore the concepts contained functions by providing dynamic teaching in accordance with the collaborative learning environment so that students could share, discuss, and restructure their knowledge in the classroom. Therefore, the CAI material has been designed to include GeoGebra activities and related student worksheets.

- In the process of developing GeoGebra-supported teaching materials, the necessary arrangements were made by taking the opinions of 5 mathematics teachers (master's students) and 3 field experts. The opinions about the instructional material introduced to the teachers were applied with the help of the evaluation form in terms of "instructional compliance" and "suitability to the curriculum". As a result of the evaluation, the majority of the teachers found the teaching material sufficient in terms of "instructional compliance" and "suitability to the curriculum". It was determined that the teachers who were interviewed had the idea that the teaching material would save students from memorization and provide a more permanent learning with the characteristics of allowing students to experiment and draw conclusions instead of directly giving the rules, concepts, formulas and relationships related to the subject. In addition, it has been determined that they visualize the concepts to be learned, that they will attract the attention of the students and that they are easy to use by the students. In addition, it has been determined that they visualize the concepts, that they will attract the attention of the students and that they are easy to use by the students.

- The pilot study of the GeoGebra-supported teaching material was conducted by the researcher with 22 seventh grade students studying in a secondary school in Isparta in the 2015-2016 academic year. GeoGebra-supported teaching activities were carried out for 3 weeks (12 lessons) with the worksheets given in Appendix 1-4.

- After the pilot application, the teaching material has been concluded via informal interviews with teachers and in-class observations.

Structure of GeoGebra Supported Teaching Material

In the scope of this research, while developing GeoGebra-supported teaching material, it has been based on Bruner's strategy of teaching through discovery and Vygosky's social integrative learning theory. In this context, GeoGebra's activities has not been presented directly to the student and thanks to the dynamic structure of GeoGebra, it has been presented the opportunity to experiment, make assumptions and discover information while accessing information. In addition, the students have been given the opportunity to share and structure their knowledge and experiences with their group friends through the worksheets.

Taking into considerations the "Circle and Disc" gains in the sub-learning area of the "Geometry and Measurement" learning area in the seventh grade mathematics curriculum of the secondary school, GeoGebra contains clear instructions that can be followed easily by teachers and students. A separate teaching material was prepared for each achievement in order to achieve meaningful learning transition between the achievements. GeoGebra screenshots has been also included in the worksheets of the teaching material considering the negativities that the students and practitioner teachers may encounter during the process. In this way, it has been aimed that

students and teachers can apply the required steps more easily. In the worksheets, the sections containing "Assessment Questions" questions related to each gain have been added to the end of the applications. These questions have been prepared by examining the curriculum and related textbooks.

In this study, GeoGebra activities developed on *"Circle and Disc"* subject. These activities were prepared for *"Determines the relations between the central angles, arcs and measurements of the circle."*, *"Calculates the length of the circle and circle piece"* and *"Calculates the area of the disc and the disc segment"* (MEB, 2013) acquisitions at the 7th grade. Detailed information about GeoGebra activities were presented below:

a) The Activity of Determining Relations Between Central Angles, Arcs and Measures in Circle

In accordance with the instructions for this acquisition, using GeoGebra with the sliding tool or by changing the location of objects (Dikovic, 2009), students were asked to determine the central angle formation in the circles shown in Figure 1, the arcs seen by the central angles, and the values whether these values change according to objects, by using the tables in the worksheet in Appendix-1. In addition, an area has been reserved where students can draw using the compasses, ruler and protractor in the radius and angle measure they want. In this way, students were able to construct geometric constructions using both DGS and concrete materials.

b) The Activity of Calculating the Length of Circle and Circle Piece

For this function, the animated activity in the screenshot in Figure 2 was used in accordance with the instructions in the worksheet in Appendix-2. In this activity, the change in the arc length due to the change in the angle value with the progress of the animation and the changes what things depending on can be observed together with the changing values. By activating the "Result" and "Generalization" buttons, it is possible to check the accuracy of the student's results.
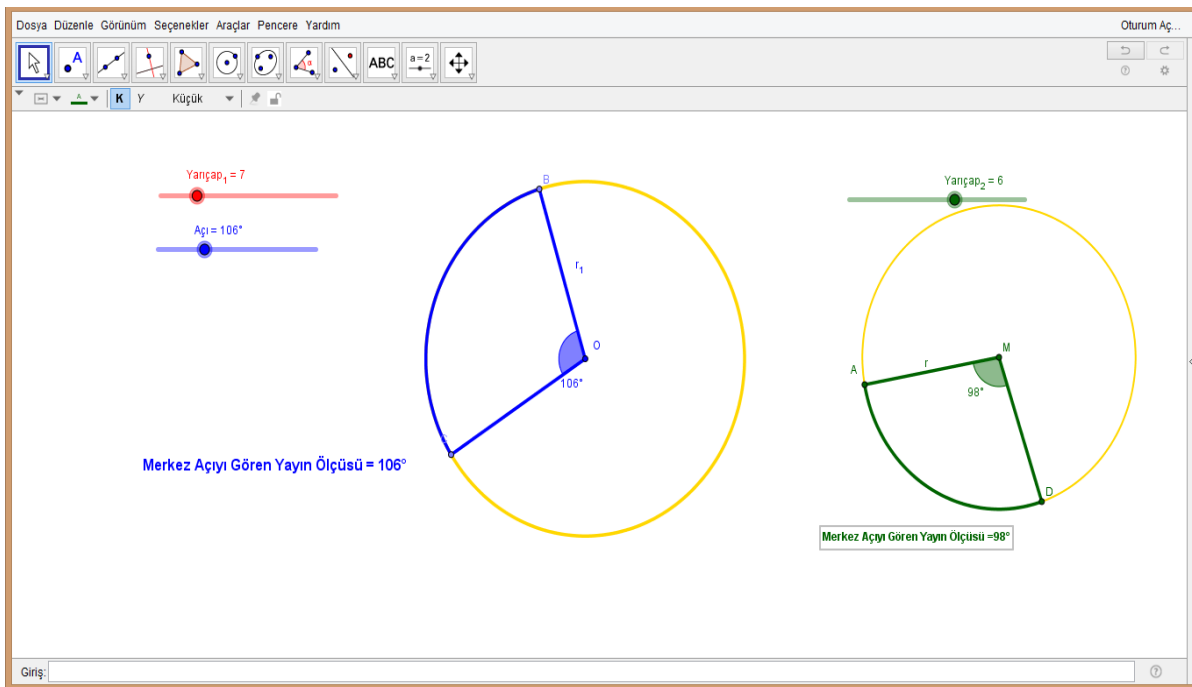
FIGURE 1. *The GeoGebra activity related to the central angle*

In the worksheet in Appendix 2, the conditions in which the length of the arc changes, the effect of the central angle and radius change on the arc length and the mathematical expression of the arc length are gradually discovered to the student. These formations, where drawing is almost impossible, can be realized thanks to the dynamics of GeoGebra. Moreover, students have the opportunity to repeat these formations as much as they want. Figure 3 shows some screenshots of this worksheet.
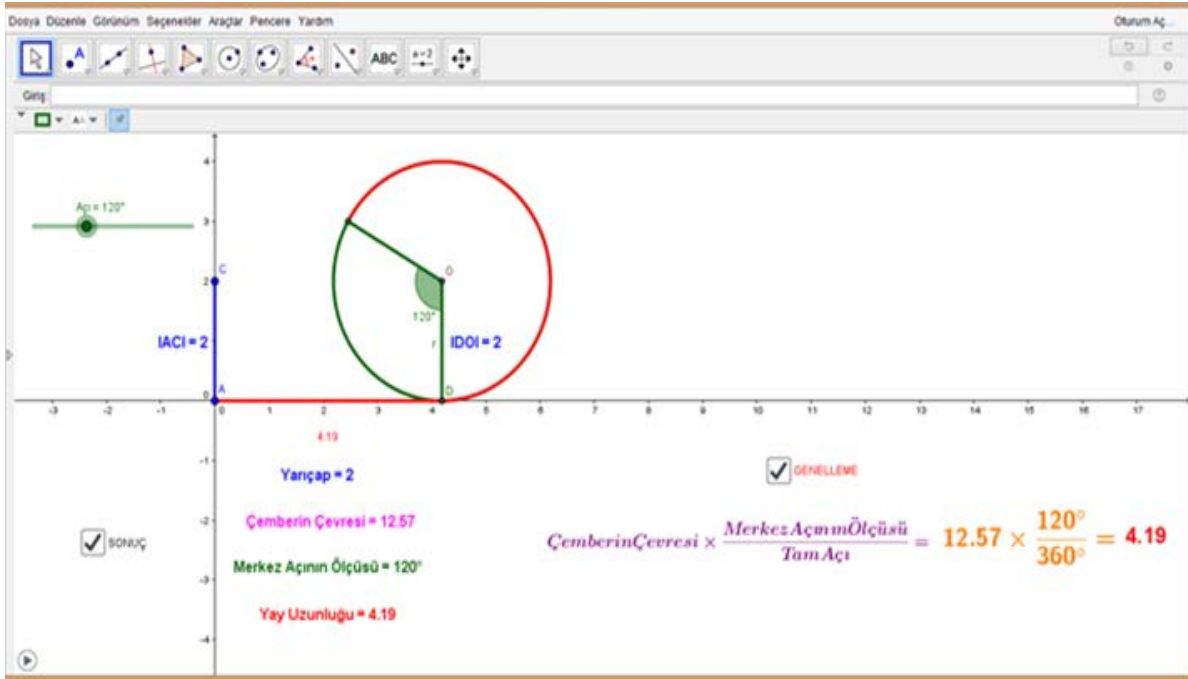
FIGURE 2. *The GeoGebra activity related to the arc length*

Follow the steps below.

1. Open the **arc length.ggb** file. Set the C point on the screen display to a radius of 2 cm.

2. By moving the angle slider, enter the arc lengths according to the central angle measure given in the table.

Radius (r)	The Measurement of the Central Angle	The Arc Length Belonging to Central Angle
2	90°	
2	180°	
2	360°	

3. Now fill in the table below for different radius values.

The Measurement of the Central Angle	Radius (r)	The Arc Length Belonging to Central Angle
90°	1	
	2	
	3	
	...	
	r	
180°	1	
	2	
	3	
	...	
	r	
360°	1	
	2	
	3	
	...	
	r	

FIGURE 3. *The screenshots of worksheet belonging to the arc length, circumference and central angle relationship*

c) The Activity of Calculating the Area of the Disc

The activities related to the function of calculating the area of the disc and the disc segment were discussed in two sections: "Calculating the area of the disc" and "Calculating the area of the disc segment". The first activity of the "Calculating the area of the disc" starts with the prediction strategy, which has an important place in mathematical thinking. As seen in the worksheet in Appendix-3, it was aimed to estimate the area of the disc with an activity with daily life content which includes estimating the coverage area of the different radio transmitters. At the end of the discovery of the area formula of the disc with the instructional material, a section has been added to this screenshot to determine the proximity of the student estimate to the real value.

In the GeoGebra activity, the screenshot of which was given in Figure 5, students were asked to complete the worksheet questions in Appendix-3 in accordance with the instructions. The activity that starts with reminding the perimeter length of the circle in Figure 4, as shown in the screenshots in Figure 5 by slicing into disc, disc turns into a parallelogram and then into a

rectangle. In this activity, students are able to explore the area of the disc by observing the transition from the area of the parallelogram and rectangle to the area of the disc, thanks to the dynamics of GeoGebra. When this transformation is achieved, by using the "divide and break" (Böl ve Parçala) buttons (Figure 4), the disc is divided into the desired part, and then with "the edit disc segments" slider the conversion can be observed between the disc and the geometric shape inserted. In this way, the teaching supported by the use of mathematics software helps students to learn and integrate mathematical knowledge with each other. As seen in the worksheet in Appendix-3, with these conversions guiding questions have been asked to discover the disc's area formula. With these questions, it is aimed for the students to express the area formula of the disc mathematically. At the end of the activity, with the "Let's Discuss" (Tartışalım) and "Result" (Sonuç) buttons shown in Figure 4, students can check the accuracy of the results they reach and discuss the results of the students in the classroom.

FIGURE 4. *The screenshots of GeoGebra activity related to the area of the disc*

(a)   (b)

FIGURE 5. *The screenshots of worksheet rerated to the parallelogram, rectangle and disc area relationship*

In addition, as seen in Figure 6 with a different GeoGebra activity, it is aimed to explore the area of the disc through the area of the triangles formed by cutting the radius of a disc formed by nested circles. In this activity, the transformations with different geometric shapes (disc-triangle-rectangle) have been observed and the changes during these transformations have been made to the students to reach the area formula of the disc. In this way, it is thought that students' ability to make assumptions can be improved by interpreting geometric shapes.

(a)   (b)

FIGURE 6. *The GeoGebra activity-2 related to the area of the disc*

The animation that starts with the movement of the "Start" (Başlat) button in the GeoGebra teaching material in Figure 6a brings about the geometric transformations in Figure 6b. The relationships, which are wanted to discover, between the area of the disc, triangle and rectangular area formed during these changes are directed through the questions in the worksheet in Appendix-3. Changes can be stopped and monitored as many times as desired. In this way, it has been thought to support the learning of students with different learning speeds. The "Undo" (Geri al) button on the screen display in Figure 6a reveals that the geometric shapes of the geometric figures formed in the worksheet (Appendix-3) are equal to each other by turning the disc back, and the students have been intended to reach the area disc formula $A = \pi.r^2$.

d) The Activity of Calculating the Area of the Disc Segment

For teaching calculation of the area of the disc segment, the worksheets in Appendix-4 contain instructions for GeoGebra activity (Figure 7) depending on the values of sizes and angles of the pizza slices that may be of interest to the students. It has been aimed that connections between segments and angles relating to segments can be identified by students, by means of visually of GeoGebra and instant observing of variable values with this activity. With this way, before area formulas of segments were found out, it was desired to provide that students could predict about disc segment's areas in mentally calculated angles such as especially $30^0, 45^0, 60^0, 90^0, 120^0, 180^0,$ 270^0 with the help of changes at the shapes.

FIGURE 7. *The GeoGebra activity for the central angle and the segment relationship*

(a)

(b)

FIGURE 8. *The screenshots of worksheet and GeoGebra related to the area of the segment of the disc*

Students were asked to complete in the table with data relating to students' observations and measurements in accordance with instructions in the worksheet and Geogebra activity (Figure 8a). They were asked to discuss about how data in the table changed for the variable radius values (Figure 8b) in the activity. Students were supposed to generalize segment's area formula for all the disc segments with the help of data in the way. In the end of the activity, it was provided that acquired results were compared with "Generalisation" button in GeoGebra activity. By means of instructions, students were asked to fill in the table at the worksheet.

DISCUSSION and CONCLUSION

In this research, GeoGebra-supported teaching material, which was developed for the "Circle and Disc" subject at the seventh grade of secondary school and pilot application was done, has been introduced. It has been concluded from the opinions of the teachers about the teaching material developed and the classroom observations in the pilot application that the GeoGebra supported teaching material is visually rich and understandable, allows the discovery of mathematical relationships and conceptions, attract the attention of the student and make the lesson enjoyable. In addition, teachers stated that the teaching material was able to create an

interesting learning environment and enable students to actively participate in the application, and it can contribute to permanent learning by concretizing the concepts. Moreover, it has been determined that the dynamic structure of the developed teaching material helps students to explore the relations by providing them with the opportunity to experiment as much as they want. Many studies (Açıkgül & Aslaner, 2015; Baltacı, Yıldız & Kösa, 2015; Kurtuluş & Uygan, 2016; Kutluca & Zengin, 2011) have highlighted similar conclusions regarding GeoGebra-supported teaching materials. The teaching material has been emphasized that from pilot application and teachers' views, it is convenient for the structure of taught subject, it can motivate students to the subject, help permanent learning and it can affect the students' attitude towards geometry positively. Teaching material has been considered that development of this material intended for making the subject find out as suitable for cooperative learning environment can contribute students' query skills positively. As a matter of fact that in literature GeoGebra-supported teaching materials have been stated that they have positive effects to cooperative learning environment (Er & Sağlam Kaya, 2017; Granberga & Olsson, 2015; Kağızmanlı & Tatar, 2016; Takači, Stankov & Milanovic, 2015; Zengin & Tatar, 2015). Improvement of these kinds of computer-assisted teaching materials will contribute to meet an important requirement in our education system. Therefore, it is suggested that computer-assisted teaching materials have been developed for teaching of different subjects and concepts in mathematics and geometry.

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Appendix-1: The example of Worksheet-1

Lesson	: Maths
Grade	: 7
Learning Area	: Geometry and Measurement
Sub-Learning Area	: Circle and Disc
Gain 1	: Determines the relations between the central angles, arcs and measurements.
Gain 2	: Calculates the length of the circle and the part of the circle.
Gain 3	: Calculates the area of the disc and disc segment.
Duration	: 10 hours

Learning-Teaching Process

Learning Output 1: Determines the relations between the central angles, arcs and measurements.

Follow the steps below.

1. Open the file **angles1.ggb**. Move the radius slider by activating the "**Move**" button and note the values taken by the angle and the length of the arc for the changing values of the radius.

Radius (r)	The Measurement of the Angle	The Length of the Arc
1		
2		
3		
...	...	...
30		

How does the angle measure and the length of the arc change as the radius increases according to your values in the table?

2. Move the angle slider by pressing the "**Move**" button. Discuss the relationship between the angle measure and the varying values of the arc length with your friends.

- Tell us what the change in angle depends on.
- Considering the position of that point, let's find a name for the angle O that sees the BC arc.
- Draw using the compasses, ruler and protractor at the desired radius and angle.
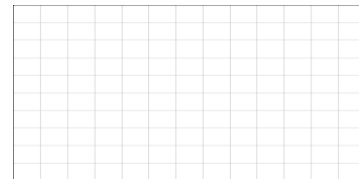
- Do your results apply to all circles? Let's answer by filling in the table below.

3. Note the values that the angle takes for the changed values of the radius by activating the "**Move**" button, moving the radius slider.

Radius (r)	The Value of the Angle
1	
2	
3	
...	...
30	

- How did the radius of the circle affect the measurement of the angle according to your values in the table?

Assessment Questions

1. Form the central angles using the points provided on the circle centered on the O. Write the center angles you create and the arcs they see with their symbols.
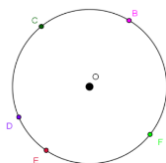
6. The numerical relationship between the number of arcs specified in the circle and the measurements of these arcs is indicated by 10.n. If n is the publication number and $m(\widehat{AB}) = 10° n$ the expression, $m(\widehat{FGH})$ how many degrees?
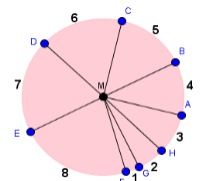
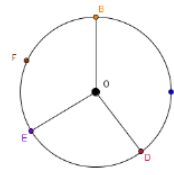

2. If the size of the CD arc (3x-10) ° is x =?
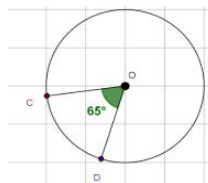
7. What is the angle between the minute hand and the minute hand when the minute hand turns 180 °?
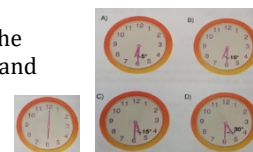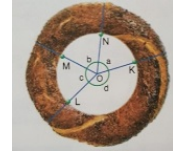
3. Specify the central angles given in the circle M.

8. Four people share a circle-shaped bagel. In this way, O is the center and 2a = 2b = 2c = d. Accordingly, what is the sum of the measurements of the KL and LM arcs?
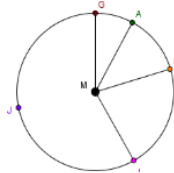
4. Find the arc length centered in the arc O centered circle (DOB) and measure the central angle of the BFE arc.

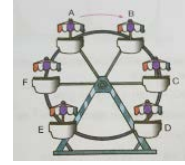**9.** In the ferris wheel shown in the figure, the cabinets are arranged at equal intervals on the circle. How many degrees of rotation in the direction of the arrow in the basket at point A will come to point E?

5. Draw a clock that shows 15:00 and find the measurement of the arc that the small angle between the scorpion and the minute hand.

10.

a. In the O-centered circle in the figure, $m(\overset{\frown}{BOA})$= 30°, what is the extent of the AB arc?
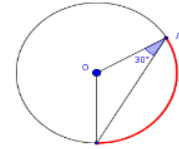
b. In the O-centered circle in the figure, the size of the AB arc is 5x + 5 and $m(\overset{\frown}{AOB})$ = 65°. Find the value x.
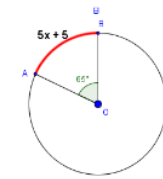
Apendix-2: The example of Worksheet-2

Learning Output 2: Calculates the length of the circle and the part of the circle.

Follow the steps below.

1. **Open the Circumference.ggb file.** What does the change in animation remind you of?
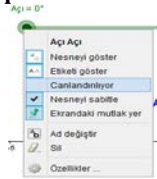
2. Stop the animation by right-clicking on the angle slider and canceling the **Animating tab.**

3. When we move the point C and make the circle 1 complete cycle with the radius values in the table, fill the table with the **IADI =** values depending on the changing radius values (use the calculator for column 3).

Radius (r)	Circumference of the Circle	Circumference / Diameter
1		
2		
3		
...	...	
r		

4. You must have remembered the custom name of the value obtained from **Circumference / Diameter** ratio from your previous information ☺.

Evaluation Questions

1. Using compasses and ruler; Draw a circle with a radius of 8 cm. Show the center, diameter and radius of the circle you draw on the shape you draw. Show the areas of the circle with points you can give examples.

2. Find the circumference of the circle with a diameter of 300 cm (π = 3.14).

5. What is the radius of the runway since an athlete who makes 5 laps around a circle-shaped race track is 1200 m? (π = 3).

6. Find the circumference of the largest area where the cow can graze by tying a 6 m long rope for grazing (π = 3).
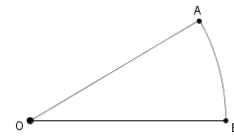
3. How many cm is the circumference of the circle with a radius of 2 cm longer than the circumference of the circle with a radius of 1 cm? (π = 3).
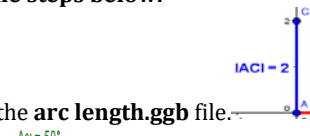
4. Find the circumference of the circle with a diameter of 35 cm (π = 22/7).

7. Find the radius of the circle with a circumference of 6π cm.

8. Since the IOAI = 9 cm and the length of the AB arc is 18π cm in the circle centered O on the side, $_m\left(\widehat{AOB}\right)$ = ?

Follow the steps below.

1. Open the **arc length.ggb** file. Set the C point on the screen display to a radius of 2 cm.

1. By moving the angle slider, enter the arc lengths according to the central angle measure given in the table.

Radius (r)	The Measurement of the Central Angle	The Arc Length Belonging to Central Angle
2	90°	
2	180°	
2	360°	

3. Now fill in the table below for different radius values.

- What is the relationship between the measurement of the central angle and the length of the arc?

- What is the relationship between the circumference of the circle and the arc length of the central angle?

- Fill in the table below using the information you find (Use the same r value).

The Measurement of the Central Angle	Radius (r)	The Arc Length Belonging to Central Angle
	1	
	2	
	3	
90°	...	
	r	
	1	
	2	
	3	
180°	...	
	r	
	1	
	2	
	3	
	...	
360°	r	

**4.** ☐ SONUÇ ☐ GENELLEME Check your results by ticking the **"Result"** and **"Generalization"** buttons.

| Radius (r)	The Measurement of the Central Angle (α)	The Measurement of the Central Angle / 360°	The Arc Length Belonging to Central Angle	Generalization
	30°			
	45°			
	60°			
	90°			
	120°			
	180°			
	240°			
	270°			
	360°			

Assessment Questions

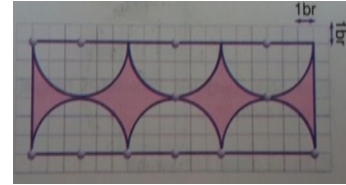**1.** Find the circumferences of the circles given below and the given arc lengths ($\pi = 3.14$).
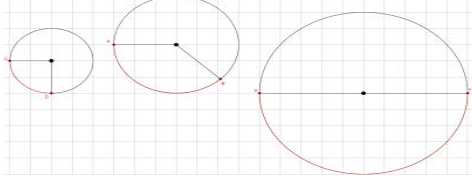
2. In the following figüre, model cars which raced in the speedway shaped with a circle having O centered and 30 m radius has been showed. Let's calculate speedway distances between racing cars if m(BOC)=60°, m(COD) = 90°, m(DOA)= 120° ve m(AOB) =90° ($\pi = 3$).
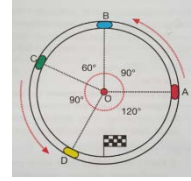
3. Find length of the full, half and quarter circle wires below ($\pi = 3$).
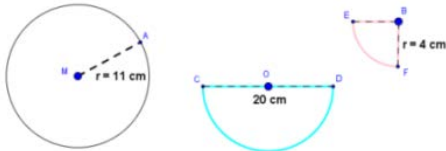
4. The figure shows a carpet pattern. What is the sum of the circumference of the colored regions? ($\pi = 3$)

5. If the circumference of the ABCD frame given below is 224 cm, find the circumference of the circle ($\pi = 22/7$).
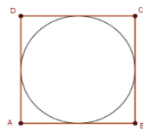
6. Find the ratio of the length of the IN arc to the length of the SL arc according to the given M centered circle segments ($\pi = 3$).
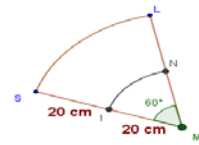
7. Find the ratio of the length of the FGH arc to the length of the FKH arc according to the O-central circle given below.


8. For a bicycle with a radius of the front wheel of 8 cm and a radius of the rear wheel of 5 cm, answer the following questions ($\pi = 3$).
- Find the perimeter of the colored circle on the rear wheel.
- Find the perimeter of the colored circle on the front wheel.
- When the colored wheel on the rear wheel rotates 4 turns, the colored wheel on the front wheel rotates 2.5 turns.
- When the colored circle on the front wheel rotates 3 turns, the circle on the rear wheel rotates 3.5 turns.

9. The front wiper of a car, 60 cm in length, operates as shown. Accordingly, how many cm will this squeegee have in total from 1 to 2? ($\pi = 3$)

10. How many centimeters would a bicycle wheel with a radius of 30 cm travel half a turn?
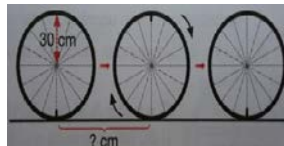
11. The triangle in the figure is an equilateral triangle. Find the measurement of the YZ arc of the circle to be drawn so that the corner M of this triangle is the center ($\pi = 3$).
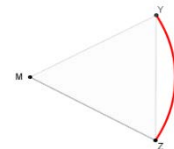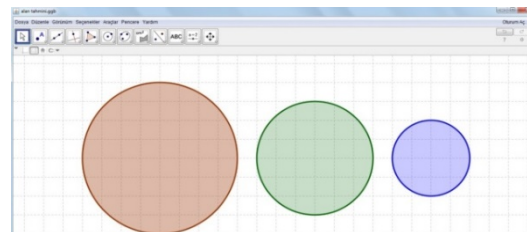

Apendix-3: The example of Worksheet-3

Learning Output 3: Calculates the area of the disc and disc segment.

The Activity-1
Follow the steps below.

1. The following screenshot shows the range of a radio transmitter. How do you estimate the range of this transmitter? Make a note.


2. Open the **activity1.ggb** file. Move the slider.

Başla
- Note the following screenshot on how we can express this length mathematically by interpreting the following change caused by the movement of the **"Start"** slider. Check the accuracy of your answer with the length button.
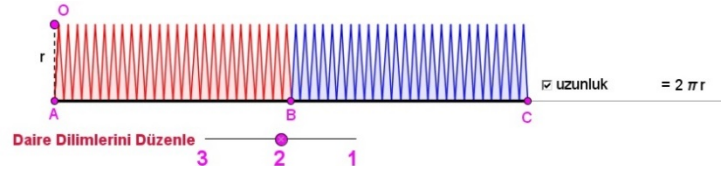
☐ böl   ☐ :)

3.   Check the **"Divide"** buttons. You will see that the circle is divided into 6 parts as shown in the figure on the right.

Daire Dilimlerini Düzenle ——————
3   2   1

Daire Dilimlerini Düzenle ——————
3   2   1
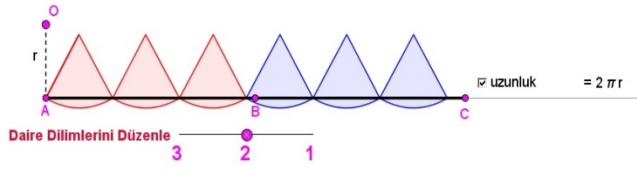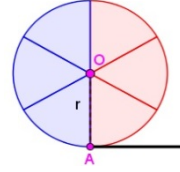
4.   Move the slider to position 2.

Parçala

5. ●———— Let's increase the number of parts by moving the slider.

Daire Dilimlerini Düzenle ——————
3   2   1

6. Discuss these two views with your friends.

Daire Dilimlerini Düzenle ——————
3   2   1

7.   Move the slider to position 1.

Parçala
●————

-      Observe the movement by dividing the disc into different parts by moving the slider.
-    Here are some screenshots. Discuss changes in mobiles with your friends.

Daire Dilimlerini Düzenle ——————
3   2   1

Daire Dilimlerini Düzenle ——————
3   2   1

Daire Dilimlerini Düzenle ——————
3   2   1

**Daire Dilimlerini Düzenle** ——————
3   2   1
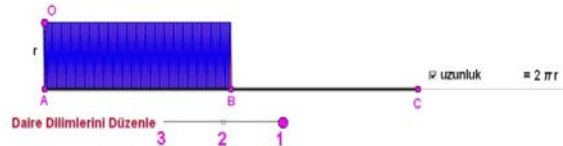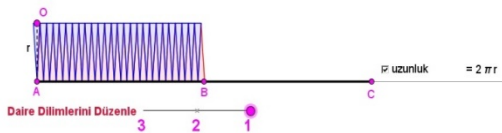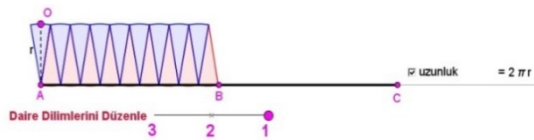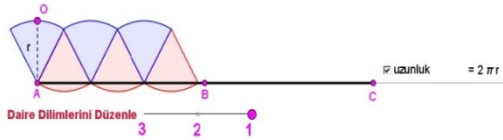
-    What are the dimensions of the resulting shapes? Express mathematically.
-    How do you calculate the areas of the resulting shapes?
-    How can we relate the areas of these geometric shapes to the area of the circle? What did you notice?

☐ Tartışalım   and   ☐ Sonuç   Make your checks with **"Let's Discuss"** and **"Result"** buttons.

The Activity-2. Open the **activityarea2.ggb** file.

8. [Başlat] Observe the movement using the **"Start"** button.

-    What geometric shape reminds you of the new formation above?
-    How can we relate this geometric shape to the disc?
-    What features do they resemble?
-    Note the mathematical expressions of the dimensions of the figure in the screenshot above.
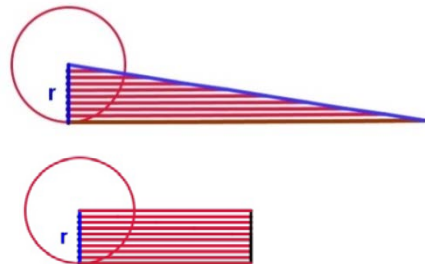
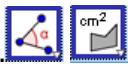-   Note the area of the figure in the screenshot above, using the dimensions.

9. [Üçgen ●——— Dikdörtgen] Move the **"Triangle"** and **"Rectangle"** slider to change the shape from triangle to rectangle.
-    What are the characteristics of a circle and a rectangle?
-    Note the mathematical expressions of the dimensions of the rectangle in the screenshot above.
-    Using the dimensions of the rectangle, note its area in the screenshot above.

10. Geri al Use the **"Undo"** button to watch the shape transform into a circle.

- How can we relate our results to the circle? Write down your thoughts by discussing with your friends.
- Calculate the coverage areas in the first activity with the area formula you discovered. Compare the results with your estimates.
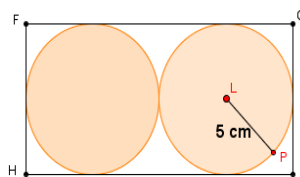
Open the **areaestimation.ggb** file. Check the results by calculating the coverage areas with the help of the **"Area"** button in the sub-menu of the **"Angle"** button.

Assessment Questions

1. Calculate the areas of the following O, M and T centered circles (π = 3).

2. Equivalent discs are cut from the rectangle given on the side. Find the sum of the non-shaded areas (π = 3).

3. A pool with a radius of 100 m will be built in the center of the Y-centered circle with a radius of 1000 m and the remaining green area will be afforested and turned into a park. Calculate the green area accordingly (π = 3.14)
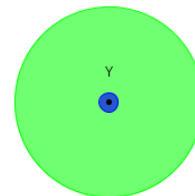
Apendix-4: The example of Worksheet-4

Follow the steps below.
1. Open the **pizzaslices.ggb** file.

Using pizza the sliders on the side, discuss the size and angle of the slices, depending on the changing values.

KalanDilimSayısı = 1

DilimSayısı = 4

yarıçap = 8

Follow the steps below.
1. Open the **segmentarea.ggb** file.

Yarıçap = 0

Using the radius slider, specify the cake in the figure according to the desired radius.

- Divide the cake into slices (sectors) using the center angles specified in the table below.

| Central Angle (α)	The Area of Cake Piece	The Area of Cake Piece / Whole Area	The Measurement of the Central Angle /360°	Generalization
30°		≅		
45°				
60°				The Area of Circle Segment =
90°				
120°				
180°				
240°				
270°				
360°				

- ☐ Dilim Fill the table with the areas of the cake slices and other desired ones by checking the **"Segment"** button.

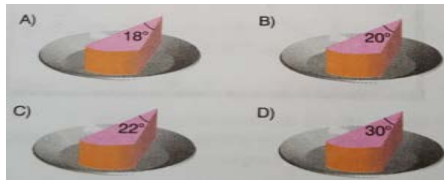☐ GENELLEME Check your results by checking the **"Generalization"** button.

Assessment Questions

1. Find the areas of the disc segments below. ($\pi = 3$).
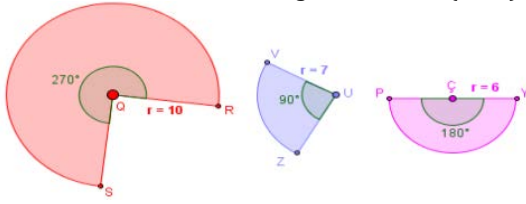
2. The circular pastry shown in the figure will be divided equally between 18 persons. Accordingly, which of the following is a piece of cake per person?


A) 18° B) 20° C) 22° D) 30°

3. Eren 6, Semih 8 and Can 4 divided 3 pizzas of equal size. The remaining slices of these three friends after eating their pizzas are given in the form. Accordingly, which of the following statements is incorrect? ($\pi = 3$).

Eren'in Pizzası   Semih'in Pizzası   Can'in Pizzası

A) Eren ate the most pizza.
B) Can and Semih ate equal amounts of pizza.
C) Can have eaten at least pizza.
D) The area of the pizza that is not eaten by Can is larger than Eren's.

4. The total area of the two discs, one of which is 9 times the other, is 270 cm². What is the radius of the small disc? ($\pi = 3$)

5. The TCD circle segment is cut and discarded from the given T-center disc segment. Find the area of the remaining region ($\pi = 3$).
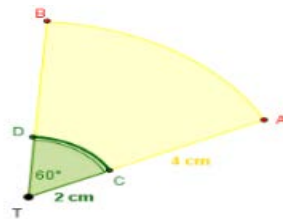

6. When a fan is turned on, it takes the form of a semi-disc with a radius of 15 cm. The areas of the disc segments that make up this range are equal. What is the area of a disc segment? ($\Pi = 3$).

7. 1 slice of pizza divided into 10 pieces with a radius of 10 cm was eaten. Accordingly, how many cm² is the area of the remaining pizza? ($\Pi = 3$).

8. The circular pizza in the figure is divided into slices of equal size and some slices are defeated. Identify the percentage of the whole pizza and the pizza with the fractions representing the missing pieces.

$\frac{3}{8}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
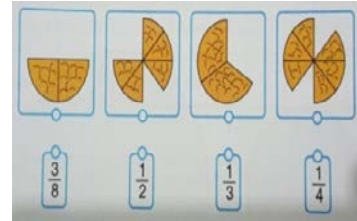
9. The figure below shows the circles centered on O_1, O_2 and O_3. Since $|O_1A| = 12\ cm$, $|O_2A| = 14\ cm$ ve $|O_3A| = 16\ cm$, how many cm² is the shaded area? ($\pi = 3$).
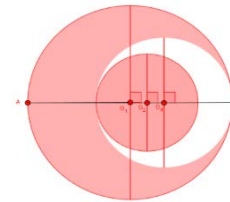
10. In a wall clock showing 2 pm, the length of the scorpion is 3 cm and the minute hand is 4 cm. According to this, how many cm² is the total area scanned by scorpion and minute hand after 2 hours? ($\pi = 3$).

11. Since the side length of each square is 1 cm, find the circumference and area of the colored part ($\pi = 3$).