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Human Capital Accumulation In Two Sector Endogenous Growth Model: Special Emphasis On Service Good – A Comparative Static Analysis

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Abstract: The present paper considers endogenous growth model with commodity sector and service sector. This extension of Lucas model shows that there exists unique steady state growth rate of human capital accumulation which works as the main driving force for the economy as a whole. On the base of an endogenous growth model, a few the comparative static analysis has been done in this paper. It shows how the share of unskilled labor force, devoted into the commodity sector is influenced by the two factors: the intensity of preference for commodity output and the service output elasticity of skilled labour.

Section 1: Introduction

The paper tries to extend the paper written by Lucas (1988), by incorporating service sector in the model where the households consume the service good. In last few decades, growth of this sector as well as that of the entire economy depends on the accumulation of human capital. There exists a number of papers that deal with the problems of service sector. A huge literature exists that modified the simple one sector Solow-Swan growth model and discussed different issues in the theory of economic growth. Studies by Uzawa (1961, 1963, 1965), Hahn (1965), Takayama (1963, 1965), Drandakis (1963), Inada (1963) introduced real balance effect in the savings function of the Solow (1956) model where the rate of savings was exogenously given. Cass (1965) and Koopmans (1965), following Ramsey (1928) model introduced household's inter temporal utility maximization behaviour in their model and endogenized the saving rate.

Empirical work includes the papers by Chand (1983), Lorence (1991), Lee and Wolpin (2006), Kirn (1987), Wood (1990), Marks (2006). Eswaran and Kotwal (2002), Sasaki (2007), Konan and Maskus (2006), Baumol et al. (1989), Pugno (2006), Zhang (2013) have done a few theoretical works on service sector. But none of these papers consider the issue of service sector and endogenous growth with Lucas (1988) type human capital accumulation function which is the main issue of the present analysis.

In a paper titled "Service Sector, Human Capital Accumulation and Endogenous Growth" written by me and my co-authors, Chakraborty et. al (2015), an endogenous growth model was formed with commodity and service sector. It derived that there exists a unique steady state growth rate of human capital accumulation which worked as the source of growth for all other sectors of the economy. On the base of the growth paths derived before, this paper tries to figure out a few comparative static analysis.The paper is organised as follows: In section 2, the basic model is presented, section 3 describes the growth rates along with the steady state equilibrium conditions and comparative static analysis and finally section 4 concludes.

2. The model:

This paper considers a closed economy model with two sectors namely, commodity sector and service sector. The total labour force is heterogeneous with respect to skill level: skilled labour and unskilled labour or raw labour. The commodity and the factor markets are characterized by perfect competition. The economy is inhabited by identical rational agents. Production technology is subject to constant returns to scale. Preferences over consumption and service streams are given by the following function where 'c' and 's' denote flow of real per capita consumption of commodity and service respectively:

$$U(c,s)=\frac{(c^{\alpha}s^{1-\alpha})^{1-\sigma}-1}{(1-\sigma)} \qquad 1.$$

Here α stands for the intensity of preference for commodity output and σ denotes the elasticity of marginal utility.

In our model, it is assumed that the output in the commodity sector can be used for consumption or investment. However, the output in the service sector is fully consumed as services cannot be stored for future uses. The commodity output is a function of human capital, raw labor and physical capital. The services are produced with human capital and raw labor only. Both the production functions are Cobb-Douglas type. Skilled labour allocates 'u' fraction of time to produce commodity output, 'v' fraction of time to produce service output and (1-u-v) fraction of time for human capital accumulation. Let 'K' be the level of physical capital in commodity sector. Let α_1 and β be commodity output elasticity of skilled labour and the service output elasticity of skilled labour respectively and α_2 be the commodity output elasticity of physical capital.The service and commodity output production functions are as follows where 'y_s' and 'y_c' be the flow of commodity output and service output respectively.

$$y_s=(N_1vh)^{\beta}((1-\phi)N_2)^{1-\beta} \qquad 2.$$

$$y_c=(N_1uh)^{\alpha_1}K^{\alpha_2}(\phi N_2)^{1-\alpha_1-\alpha_2} \qquad 3.$$

It is assumed that the number of skilled labor and unskilled labor be N_1 and N_2 respectively and N be the total labour force or working population such that

$$\mathrm{N} = \mathrm{N}_1 + \mathrm{N}_2 \qquad 4.$$

Population grows at a constant, exogenous rate and more over we assume that each segment of the population is growing at the same rate in the following way:

$$\frac{\dot{\mathrm{N}}}{\mathrm{N}} = \frac{\dot{\mathrm{N}}_1}{\mathrm{N}_1} = \frac{\dot{\mathrm{N}}_2}{\mathrm{N}_2} = \mathrm{n} \geq 0 \qquad 5.$$

It is further assumed that the general skill level of a worker is 'h'. The effective skilled work force in commodity production is '$\mathrm{N}_1\mathrm{uh}$' and that in service production is 'N_1vh'. These are skill-weighted man-hours devoted to current commodity and service output production respectively. Let 'ϕ' be the fraction of total unskilled labor force that is devoted in the commodity sector. The remaining (1- ϕ) fraction is engaged in producing service output.

Following Lucas (1988), the accumulation of human capital is assumed to be proportional to the time allocated for education. Hence, human capital accumulation function is given by

$$\gamma_{\mathrm{h}} = \dot{\mathrm{h}}/\mathrm{h} = \delta(1-\mathrm{u}-\mathrm{v}) \qquad 6.$$

Here δ be the productivity parameter in the human capital accumulation function. It is assumed that commodity output over aggregate consumption is accumulated as physical capital. The physical capital accumulation function is given by

$$\dot{K} = y_c - Nc \qquad 7.$$

Service output is totally consumed. So the market clearing condition is

$$y_s = Ns \qquad 8.$$

The objective of the economy is to maximize the value of utility defined by equation (1) subject to the constraints given by (2), (3), (6), (7) and (8).

The current value Hamiltonian of this dynamic optimization problem can be formulated as

$$H = \frac{N}{(1-\sigma)}[(c^{\alpha}s^{1-\alpha})^{1-\sigma} - 1] + \theta_1[(N_1uh)^{\alpha_1}K^{\alpha_2}(\phi N_2)^{1-\alpha_1-\alpha_2} - Nc]$$
$$+ \theta_2[\delta h(1-u-v)] + \lambda[(N_1vh)^{\beta}((1-\phi)N_2)^{1-\beta} - Ns] \qquad 9.$$

Where θ_1 and θ_2 are shadow prices associated with $\dot{K}$ and $\dot{h}$ respectively and λ be the Lagrange multiplier as the market clearing condition given by equation (9) is a static constraint. There are five control variables c, s, u, v, ϕ in this model. The state variables are K and h.

3. Steady state Growth path

The model results show that along the steady state growth path c, s, K grow at constant rate and u, v, φ are time independent. In the steady state the growth path of human capital is given in equation (10a).

$$\gamma_h = \delta(1-u-v) = \frac{A}{B} \qquad 10a.$$

Here $A = n + \delta - \rho$ 10.b

$B = 1 - x$ 10.c

Where $x = (1-\sigma)\{\beta(1-\alpha) + \frac{\alpha\alpha_1}{(1-\alpha_2)}\}$ 10d.

Let γ_i denotes the growth rate of ith variable where i= c, s, and K . Equilibrium growth path for these variables are given as follows:

$$\gamma_c = (\frac{\alpha_1}{1-\alpha_2})\gamma_h \qquad 11a.$$

$$\gamma_s = \beta\gamma_h \qquad 11b.$$

$$n + \gamma_c = \gamma_K \qquad 11c.$$

We observe that growth rate of human capital is the source of growth for other three sectors. Thus, this model with service sector where entire service output is consumed confirms the findings of Lucas model that human capital accumulation is the engine of growth.

From the first order conditions of optimization we obtain the values of u, v and ϕ defined as follows:

$$u = (\delta B - A)/\delta B(1+D) \qquad 12a.$$

$$v = D(\delta B - A)/\delta B(1+D) \qquad 12b.$$

And $\phi = \frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D + (1-\alpha_1-\alpha_2)\beta}$ 12c.

Where D is a constant and given by

$$D = \frac{(1-\alpha)\beta}{\alpha\alpha_1}[\frac{(\rho - n\alpha_2)Z - (n+\delta-\rho)(Y+\alpha_1\alpha_2)}{\rho Z - (n+\delta-\rho)Y}] \quad 12d.$$

Where $Z = (1-\alpha_2)[1-(1-\sigma)\{\beta(1-\alpha) + \frac{\alpha\alpha_1(1-\sigma)}{(1-\alpha_2)}\}]$ 12e.

And $Y = \alpha_1\{\alpha(1-\sigma)-1\} + \beta(1-\alpha)(1-\alpha_2)(1-\sigma)$ 12f.

Hence in this model the values of u, v and ϕ are endogenously determined. The model to be economically viable u, v and 1-u-v should be positive.

<u>The conditions for positive u, v and (1-u-v) are</u>:

When σ<1; the positivity of u and v requires the following condition:

$\delta B \geq A$

Or, $\delta(1-x) \geq n+\delta-\rho$

Or, $\rho \geq n+\delta x$ 13a.

From $(1-u-v) \geq 0$, we get $\frac{A}{\delta B} \geq 0$

Therefore the required conditions are: A and B must be positive. So, from the requirement that $A \geq O$ and $B \geq 0$ we get 13b. and 13c. equations respectively

$n+\delta \geq \rho$ 13b.

And $1-x \geq 0$,

Or. $x \leq 1$ 13c.

Merging equation no. 13a.and 13b we get,

$n+\delta \geq \rho \geq n+\delta x$

This is the same condition that is required for positive B. Combining extreme left and extreme right side of the above inequality we in fact have 13c.When σ≥1 : The required conditions are the same as the previous case.

Now the positivity of D is also required for positive values of u and v.

The conditions required for positive D:

When σ<1; the sufficient condition $\alpha_1 > \alpha_1^*$

Where $\alpha_1^* = \dfrac{\alpha_2 + (1-\sigma)\beta(1-\alpha)(1-\alpha_2)}{1-(1-\sigma)\alpha}$.

Thus, combining the conditions we have the following growth path:

There exists positive, unique steady state growth rate for human capital consumption service and capital goods sectors.

The detailed derivation of the growth paths has been done in the appendix of the paper written by me and my co-authors, titled "Service Sector, Human Capital Accumulation and Endogenous Growth" which was published in Theoretical and Applied Economics Volume XXII (2015), No. 4(605), Winter, pp. 199-216.

Comparative static analysis:

We have done comparative static analysis on ϕ. Here 'ϕ' is the fraction of total unskilled labor force that is devoted in the commodity sector. α stands for the intensity of preference for commodity output. β is the service output elasticity of skilled labour. Differentiating ϕ with respect to α and β we get (detailed derivation is done in appendix),

$$\frac{\partial \phi}{\partial \alpha} > 0$$

Proposition 1: When the intensity of preference for commodity output rises, the fraction of total unskilled labor force engaged in the commodity sector also rises.

The reason behind such result is that when the intensity of preference for commodity output rises, the requirement for more commodity production also rises. As a result, the fraction of total unskilled labor force engaged in the commodity sector also rises.

$$\frac{\partial \phi}{\partial \beta} > 0$$

Proposition 2: When the service output elasticity of skilled labour increases, the fraction of total unskilled labor force engaged in the commodity sector also rises.

The reason behind such result is that as the two sectors are interdependent, Whenever the relative responsiveness of service output with respect to skilled labor rises, the share of unskilled labor force engaged in the commodity sector also rises so that the remaining share of unskilled labor who are engaged in service sector, decreases. The service sector become more efficient in this manner.

5 Conclusion:

This paper develops an endogenous growth model with commodity sector and service sector with Lucas type human capital accumulation function. This extension of Lucas model shows that that there exists unique steady state growth rate of human capital accumulation which works as the main driving force for the economy as a whole. It is found from the comparative static analysis that **when the intensity of preference for commodity output rises, the fraction of total unskilled labor force engaged in the commodity sector also rises and secondly, we have found when** the service output elasticity of skilled labour increases, **the fraction of total unskilled labor force engaged in the commodity sector also rises.**

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Appendix:

$$\phi = \frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D+(1-\alpha_1-\alpha_2)\beta}$$

if $\alpha_1 > \alpha_2$ and $(n+\delta-\rho)>0$

$D = A\frac{(1-\alpha)\beta}{\alpha\alpha_1}$ Here A is not function of α and β

$$\phi = \frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D+(1-\alpha_1-\alpha_2)\beta}$$

$$D = \frac{(1-\alpha)}{\alpha\alpha_1}[\frac{(\rho-n\alpha_2)(1-\alpha_2)+(n+\delta-\rho)(1-\alpha_2)\alpha_1}{\rho(1-\alpha_2)+(n+\delta-\rho)\alpha_1}]$$

Where $Z = (1-\alpha_2)[1-(1-\sigma)\{\beta(1-\alpha)+\frac{\alpha\alpha_1(1-\sigma)}{(1-\alpha_2)}\}]$

$$Y = \alpha_1\{\alpha(1-\sigma)-1\}+\beta(1-\alpha)(1-\alpha_2)(1-\sigma)$$

When $\sigma = 1$,

$$D = \frac{(1-\alpha)\beta}{\alpha\alpha_1}[\frac{(\rho-n\alpha_2)(1-\alpha_2)+(1-\alpha_2)+(n+\delta-\rho)(1-\alpha_2)\alpha_1}{\rho(1-\alpha_2)+(n+\delta-\rho)\alpha_1}]$$

$$D = \frac{(1-\alpha)\beta}{\alpha\alpha_1}[\frac{\rho\{(1-\alpha_2)(1-\alpha_1)\}+(1-\alpha_2)n((\alpha_1-\alpha_2)+\delta\alpha_1(1-\alpha_2)}{\rho(1-\alpha_2)+(n+\delta-\rho)\alpha_1}]$$

Let $A=[\frac{\rho\{(1-\alpha_2)(1-\alpha_1)\}+(1-\alpha_2)n((\alpha_1-\alpha_2)+\delta\alpha_1(1-\alpha_2)}{\rho(1-\alpha_2)+(n+\delta-\rho)\alpha_1}]$

So, $D=A\frac{(1-\alpha)\beta}{\alpha\alpha_1}$ Here A is not function of α and β

$$\phi=\frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D+(1-\alpha_1-\alpha_2)\beta}$$

if $\alpha_1>\alpha_2$ and $(n+\delta-\rho)>0$

Here A is not function of α and β

Differentiating D with respect to α,

$$\frac{\partial D}{\partial\alpha}=-(\frac{1}{\alpha^2})[\frac{A\beta}{\alpha_1}]$$

Therefore

$$\frac{\partial D}{\partial\alpha}<0$$

Differentiating D with respect to β,

$$\frac{\partial D}{\partial\beta}=\frac{A(1-\alpha)}{\alpha\alpha_1}$$

$$\frac{\partial D}{\partial\beta}>0$$

$$\phi=\frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D+(1-\alpha_1-\alpha_2)\beta}=\frac{M}{ND+M}$$

Here $M=(1-\alpha_1-\alpha_2)\beta \quad D=f(\alpha)$

$$\frac{\partial\phi}{\partial\alpha}=\frac{(ND+M)\frac{\partial M}{\partial\alpha}-M\frac{\partial(ND+M)}{\partial\alpha}}{(ND+M)^2}$$

Now as $\frac{\partial D}{\partial\alpha}<0$, $\frac{\partial\phi}{\partial\alpha}$>0

Therefore, $\frac{\partial \phi}{\partial \alpha} > 0$

$$\phi = \frac{(1-\alpha_1-\alpha_2)\beta}{\alpha_1(1-\beta)D+(1-\alpha_1-\alpha_2)\beta}$$

Or, $\phi = \dfrac{(1-\alpha_1-\alpha_2)}{\alpha_1 \frac{(1-\beta)}{\beta} D + (1-\alpha_1-\alpha_2)}$

Now, $D = A\dfrac{(1-\alpha)\beta}{\alpha\alpha_1}$

$$\phi = \frac{1}{\frac{A(1-\alpha)(1-\beta)}{\alpha(1-\alpha_1-\alpha_2)}+1} = \frac{1}{S(1-\beta)+1} \quad \text{Where } S = \frac{A(1-\alpha)}{\alpha(1-\alpha_1-\alpha_2)}$$

Using the division rule of differentiation, we derive

$\frac{\partial \phi}{\partial \beta}$ >0 ……………………finding 2